Note ♯7 (Maximum and Minimum Values, Mean Value Theorem, How Derivatives Affect the Shape of a Graph, Indeterminate Forms and L'Hospital's Rule)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, Video List.

(1) Find the absolute maximum and absolute minimum values of f on the given interval.

(a) $f(x) = 10 + 4x - x^2, \quad [0, 5]$

key: Absol.Max=14, Absol.min=5

(b) $f(x) = (x^2 - 4)^3, \quad [-1, 3]$

key: Absol.Max=125, Absol.min=−64

(c) $f(x) = 2\cos x + \sin 2x, \quad [0, \pi/2]$

key: Absol.Max=$\frac{3\sqrt{3}}{2}$*, Absol.min=0*

(2) Find the number c that satisfies the conclusion of the Mean Value Theorem on the given interval.

(a) $f(x) = 2x^2 - 3x + 1, \quad [0, 2]$

key: $c = 1$

(b) $f(x) = \ln x, \quad [1, 4]$

key: $c = \dfrac{3}{\ln 4}$

(3) Sketch a curve satisfying the following conditions.

(a) The domain of $f(x)$ is all real numbers.

(b) $f(2) = -2$, $f(0) = 0$, $f(4) = 2$, $f'(2) = 0$.

(c) $f'(x) < 0$ if $0 < x < 2$, $f'(x) > 0$ if $x > 2$.

(d) $f''(x) < 0$ if $0 \leq x < 1$ or if $x > 4$.

(e) $f''(x) > 0$ if $1 < x < 4$.

(f) $\lim_{x \to \infty} f(x) = 2$.

(g) The graph of $f(x)$ is symmetric about the y-axis.

key: Watch the video

(4) Find the limit.

(a) $\lim_{x\to -2} \dfrac{x^3+8}{x+2}$

key: 12

(b) $\lim_{x\to \pi/2} \dfrac{1-\sin x}{1+\cos 2x}$

key: $\frac{1}{4}$

(c) $\lim_{x\to\infty} \frac{\ln\sqrt{x}}{x^2}$

key: 0

(d) $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

key: $\frac{1}{2}$

(e) $\lim_{x\to\infty} x^3 e^{-x^2}$

key: 0

(f) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

key: $\frac{1}{2}$

(g) $\lim_{x\to 1^+} x^{1/(1-x)}$

key: $\dfrac{1}{e}$

(h) $\lim_{x\to 0^+} (4x+1)^{\cot x}$

key: e^4

(5) Sketch the graph of $f(x) = \dfrac{x}{x^2 - 4}$ by locating intervals of increase/decrease, local extrema, concavity, and inflection points.

key: Watch the video