Note ♯8 (Optimization Problems, Antiderivatives, Area and Distance)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, Video List.

(1) The top and bottom margins of a poster are each $9cm$ and the side margins are each $6cm$. The area of printed material on the poster is fixed at $864cm^2$. Find the dimensions of the printed area that minimize the area of the whole poster.
key: $24\ cm \times 36\ cm$

(2) Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 6 - x^2$.

key: $2\sqrt{2} \times 4$

(3) Find the point on the line $5x + y = 7$ that is closest to the point $(-2, 1)$.

key: $\left(\frac{14}{13}, \frac{21}{13}\right)$

(4) Find the most general antiderivative of the function.

(a) $f(x) = 6x^5 - 8x^4 - 9x^2$

key: $F(x) = x^6 - \frac{8}{5}x^5 - 3x^3 + C$

(b) $f(x) = (x-5)^2$

key: $F(x) = \frac{1}{3}x^3 - 5x^2 + 25x + C$

(c) $g(t) = \dfrac{1+t+t^2}{\sqrt{t}}$

key: $G(t) = 2t^{\frac{1}{2}} + \frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + C$

(d) $g(x) = 2\cos x - \dfrac{3}{\sqrt{1-x^2}}$

key: $G(x) = 2\sin x - 3\arcsin(x) + C$ *or* $G(x) = 2\sin x + 3\arccos(x) + C$

(5) Find f.

(a) $f''(x) = 2x^3 - 12x^2 + 6x$

key: $f(x) = \frac{1}{10}x^5 - x^4 + x^3 + Cx + D$

(b) $f''(x) = 2x + 3e^x$

key: $f(x) = \frac{1}{3}x^3 + 3e^x + Cx + D$

(c) $f''(x) = 1/x^2$

key: $f(x) = -\ln|x| + Cx + D$

(d) $f'(x) = 5x^4 - 3x^2 + 4, \quad f(-1) = 2$

key: $f(x) = x^5 - x^3 + 4x + 6$

(e) $f''(\theta) = \sin\theta + \cos\theta, \quad f(0) = 3, \quad f'(0) = 4$

key: $f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4$

(f) $f''(x) = 4 + 6x + 24x^2, \quad f(0) = 3, \quad f(1) = 10$

key: $f(x) = 2x^2 + x^3 + 2x^4 + 2x + 3$

(6) A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff?

key: 225 ft

(7) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using

(a) Three rectangles and right endpoints.

key: 8

(b) Three rectangles and left endpoints.

key: 5

(c) Three rectangles and midpoints.

key: $\frac{23}{4}$

(8) Use the Definition to find an expression for the area under the graph of f as a limit.

$$f(x) = \frac{6x}{x^2+8}, \quad 1 \le x \le 3$$

key: $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \frac{6(1+\frac{2}{n}i)}{(1+\frac{2}{n}i)^2+8}$