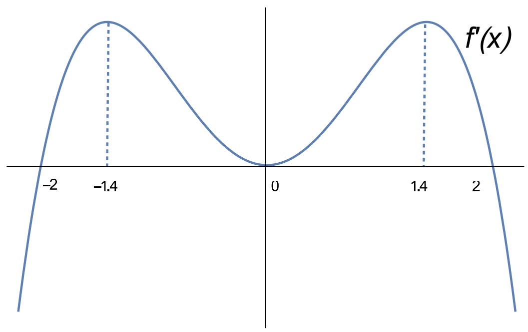
Note ♯9 (Exam3 review)

Click the boxed answer to watch the video solution. If you want to see the list of video, click this link, Video List.

(1) The graph below is that of the derivative, $f'(x)$, of a continous function $f(x)$. Find the interval(s) where $f(x)$ is increasing, decreasing, concave up, concave down, and find the x-values of local extrema and inflection point.



key: *Watch the Video*
increasing: $(-2, 0), (0, 2)$, *decreasing:* $(-\infty, -2), (2, \infty)$,
local min at $x = -2$, *local max at* $x = 2$,
concave up: $(-\infty, -1.4), (0, 1.4)$, *concave down:* $(-1.4, 0), (1.4, \infty)$,
inflection at $x = -1.4, 0, 1.4$

(2) Find the absolute maximum and minimum values for $f(x) = x^2 + \dfrac{2}{x}$ over the interval $[\frac{1}{2}, 2]$.

key: Absolute Max = 5, Absolute min = 3

(3) If $f'(x) = (x-2)^3(4-x)^5(x+1)^2$, find the interval(s) where $f(x)$ is increasing and decreasing.

key: increasing: $(2,4)$, *decreasing:* $(-\infty,-1),(-1,2),(4,\infty)$

(4) If $f(x) = \dfrac{x^2}{x-1}$, find the interval(s) where $f(x)$ is increasing and decreasing, and find the x-value(s) of local maximum and minimum.

key: Watch the Video *increasing:* $(-\infty, 0), (2, \infty)$
decreasing: $(0, 1), (1, 2)$
local max at $x = 0$
local min at $x = 2$

(5) If $f(x) = x^4 - 6x^2 + 4$, find the interval(s) where $f(x)$ is increasing, decreasing, concave up, and concave down. And find the local extream and inflection point(s).

key: Watch the Video *increasing:* $(-\sqrt{3}, 0), (\sqrt{3}, \infty)$,
decreasing: $(-\infty, -\sqrt{3}), (0, \sqrt{3})$,
local Max: $(0, 4)$, *local min:* $(-\sqrt{3}, -5), (\sqrt{3}, -5)$,
concave up: $(-\infty, -1), (1, \infty)$, *concave down:* $(-1, 1)$,
inflection point: $(-1, 2), (1, 2)$

(6) If $f(x) = 12 - x - \dfrac{9}{x}$, find the interval(s) where $f(x)$ is increasing, decreasing, concave up, and concave down. And find the local extrema and inflection point(s).

key: Watch the Video *increasing:* $(-3, 0), (0, 3)$, *decreasing:* $(-\infty, -3), (3, \infty)$,
local min: $(-3, 18)$, *local max:* $(3, 6)$,
concave up: $(-\infty, 0)$, *concave down:* $(0, \infty)$,
no inflection point

(7) Consider the function $f(x) = \dfrac{(x+2)^3}{(x-1)^2}$ for which $f'(x) = \dfrac{(x+2)^2(x-7)}{(x-1)^3}$ and $f''(x) = \dfrac{54(x+2)}{(x-1)^4}$. Find the interval(s) where $f(x)$ is increasing, decreasing, concave up and down. And find the local extrema and inflection point(s).

key: Watch the Video *increasing:* $(-\infty,-2),(-2,1),(7,\infty)$, *decreasing:* $(1,7)$,
local min: $(7,81/4)$,
concave up: $(-2,1),(1,\infty)$, *concave down:* $(-\infty,-2)$,
inflection: $(-2,0)$

(8) Evaluate

(a) $\lim_{x\to\infty} [\ln(2+3x) - \ln(4+5x)]$

key: $\ln(3/5)$

(b) $\lim_{x\to\infty} \left(\frac{2x^2}{2x+1} - \frac{x^2}{x+3} \right)$

key: $= 5/2$

(c) $\lim_{x\to 1} \dfrac{e^{2x-2}+x^2-2}{\ln x + 2x - 2}$

key: $= 4/3$

(d) $\lim_{x\to\infty} (1+x+x^2)^{\frac{1}{\ln x}}$

key: $= e^2$

(e) $\lim_{x\to\infty} x\sin(\pi/x)$

key: π

(f) $\lim_{x\to 0}(1-5x)^{\frac{1}{x}}$

key: $= \dfrac{1}{e^5}$

(9) Find the number c that satisfies the conclusion of Mean Value Theorem on the given interval.

$$f(x) = (x-1)^{10}, \quad [0,2]$$

key: 1

(10) Find $f(x)$

(a) $f'(x) = x^2(x^3 + 1)$

key: $f(x) = \frac{1}{6}x^6 + \frac{1}{3}x^3 + C$

(b) $f'(x) = \dfrac{3x^4 + 1}{x^5}$

key: $f(x) = 3\ln|x| - \dfrac{1}{4x^4} + C$

(c) $f'(x) = \sec x(\sec x + \tan x)$

key: $f(x) = \tan x + \sec x + C$

(d) $f'(x) = 3x^2 + 4\sin x + e^x, \quad f(0) = 10$

key: $f(x) = x^3 - 4\cos x + e^x + 13$

(e) $f''(x) = 8x^3 + 5, \quad f(1) = 0, \ f'(1) = 8$

key: $f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x - \frac{39}{10}$

(f) $f''(x) = 4 + 6x + 24x^2, \quad f(0) = 3, \ f(1) = 10$

key: $f(x) = 2x^2 + x^3 + 2x^4 + 2x + 3$

(11) Approximate the area under the graph of $f(x) = 20 - x^2$ from $x = -2$ to $x = 4$ using 6 equal width subintervals and using right endpoints.

key: 89

(12) Approximate the area under the graph of $f(x) = 2\sqrt{x}$ from $x = 1$ to $x = 4$ using 3 equal width subintervals and using left endpoints.

key: $2 + 2\sqrt{2} + 2\sqrt{3}$

(13) Use Definition of area to find an expression for the area under the graph of $f(x) = \dfrac{x^2}{x-3}$ in $[1, 3]$ as a limit.

key: $A = \lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{(1 + \frac{2}{n}i)^2}{(1 + \frac{2}{n}i) - 3} \cdot \dfrac{2}{n}$

(14) Use geometry to evaluate the following integrals.

(a) $\int_{-1}^{2} (1-x)\,dx$

key: $3/2$

(b) $\int_{0}^{9} (\frac{1}{3}x - 2)\,dx$

key: $-9/2$