Math 251 - Week-In-Review 1

Justin Cantu

Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. What does $y = 6 - z$ represent in $\mathbb{R}^3$?
2. Find the center and radius of the sphere
$$x^2 + y^2 + z^2 - x + 4y - 10z = 1$$
3. Find an equation of the sphere if one of its diameters has endpoints $(2, 1, -3)$ and $(8, 3, 5)$.
4. Given the points $A(-2, 4, 2)$ and $B(3, 0, 8)$, find $\overrightarrow{AB}$.
5. Given $\mathbf{a} = \langle 1, -1, 5\rangle$ and $\mathbf{b} = \langle -3, 2, 1\rangle$, find $|\mathbf{a} + 2\mathbf{b}|$.
6. Find the vector that has the same direction as $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ but has length 5.
7. Compute $\mathbf{a} \cdot \mathbf{b}$ if
   (a) $\mathbf{a} = \langle 11, -2, -1\rangle$ and $\mathbf{b} = \langle 3, 4, -7\rangle$.
   (b) $|\mathbf{a}| = 5$, $|\mathbf{b}| = 2$, and $\theta = 30°$.
8. Find the angle between the vectors $\mathbf{a} = \langle 4, 2, -1\rangle$ and $\mathbf{b} = \langle -3, 0, 4\rangle$.
9. The points $A(5, 1, 7)$, $B(1, -2, 3)$, and $C(2, -1, 6)$ form a triangle. Find $\angle ABC$.
10. Find the scalar and vector projections of $\langle 2, 4, 6\rangle$ onto $\langle 1, 3, 5\rangle$.
11. If $\mathbf{a} = \langle 3, -1, 7\rangle$ and $\mathbf{b} = \langle 2, 2, 5\rangle$, find $\mathbf{b} \times \mathbf{a}$.
12. Find a nonzero vector that is orthogonal to the plane passing through the points $P(0, 1, 2)$, $Q(4, -2, 5)$, and $R(5, 4, -3)$.
13. Find the area of the parallelogram determined by $\mathbf{a} = \langle 3, 0, 2\rangle$ and $\mathbf{b} = \langle 1, -4, 5\rangle$.
14. Find $|\mathbf{a} \times \mathbf{b}|$ if $|\mathbf{a}| = 6$, $|\mathbf{b}| = 2$, and $\theta = 3\pi/4$.

Solutions

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 1 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. What does $y = 6 - z$ represent in $\mathbb{R}^3$?

   A plane (see video for sketch).

2. Find the center and radius of the sphere

$$x^2 + y^2 + z^2 - x + 4y - 10z = 1$$

   $C\left(\frac{1}{2}, -2, 5\right), \quad r = \frac{11}{2}$

3. Find an equation of the sphere if one of its diameters has endpoints $(2, 1, -3)$ and $(8, 3, 5)$.

   $(x-5)^2 + (y-2)^2 + (z-1)^2 = 26$

4. Given the points $A(-2, 4, 2)$ and $B(3, 0, 8)$, find $\overrightarrow{AB}$.

   $\overrightarrow{AB} = \langle 5, -4, 6 \rangle$

5. Given $\mathbf{a} = \langle 1, -1, 5 \rangle$ and $\mathbf{b} = \langle -3, 2, 1 \rangle$, find $|\mathbf{a} + 2\mathbf{b}|$.

   $|\mathbf{a} + 2\mathbf{b}| = \sqrt{83}$

6. Find the vector that has the same direction as $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ but has length 5.

   $\frac{10}{3}\mathbf{i} + \frac{5}{3}\mathbf{j} - \frac{10}{3}\mathbf{k}$ or $\left\langle \frac{10}{3}, \frac{5}{3}, -\frac{10}{3} \right\rangle$

7. Compute $\mathbf{a} \cdot \mathbf{b}$ if

   (a) $\mathbf{a} = \langle 11, -2, -1 \rangle$ and $\mathbf{b} = \langle 3, 4, -7 \rangle$. $\mathbf{a} \cdot \mathbf{b} = 32$

   (b) $|\mathbf{a}| = 5$, $|\mathbf{b}| = 2$, and $\theta = 30°$. $\mathbf{a} \cdot \mathbf{b} = 5\sqrt{3}$

8. Find the angle between the vectors $\mathbf{a} = \langle 4, 2, -1 \rangle$ and $\mathbf{b} = \langle -3, 0, 4 \rangle$.

   $\theta = \cos^{-1}\left(\dfrac{-16}{5\sqrt{21}}\right)$

9. The points $A(5, 1, 7)$, $B(1, -2, 3)$, and $C(2, -1, 6)$ form a triangle. Find $\angle ABC$.

   $\theta = \cos^{-1}\left(\dfrac{19}{\sqrt{41}\sqrt{11}}\right)$

10. Find the scalar and vector projections of $\langle 2, 4, 6 \rangle$ onto $\langle 1, 3, 5 \rangle$.

    $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{44}{\sqrt{35}}, \quad \text{proj}_{\mathbf{a}}\mathbf{b} = \left\langle \dfrac{44}{35}, \dfrac{132}{35}, \dfrac{44}{7} \right\rangle$

11. If $\mathbf{a} = \langle 3, -1, 7 \rangle$ and $\mathbf{b} = \langle 2, 2, 5 \rangle$, find $\mathbf{b} \times \mathbf{a}$.

    $\mathbf{b} \times \mathbf{a} = \langle 19, 1, -8 \rangle$

12. Find a nonzero vector that is orthogonal to the plane passing through the points $P(0, 1, 2)$, $Q(4, -2, 5)$, and $R(5, 4, -3)$.

    $\langle 6, 35, 27 \rangle$ (or some scalar multiple of this, e.g., $\langle -6, -35, -27 \rangle$)

13. Find the area of the parallelogram determined by $\mathbf{a} = \langle 3, 0, 2 \rangle$ and $\mathbf{b} = \langle 1, -4, 5 \rangle$.

    The area is $\sqrt{377}$.

14. Find $|\mathbf{a} \times \mathbf{b}|$ if $|\mathbf{a}| = 6$, $|\mathbf{b}| = 2$, and $\theta = 3\pi/4$.

    $|\mathbf{a} \times \mathbf{b}| = 6\sqrt{2}$