Math 251 - Week-In-Review 10

Justin Cantu

Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Evaluate $\iint_S (z + xy)\, dS$, where S is the part of the cylinder $x^2 + y^2 = 4$ that lies between the planes $z = 0$ and $z = 4$.

2. Evaluate $\iint_S xz\, dS$, where S is the surface $y = 2x + z^2$, $2 \le x \le 4$, $0 \le z \le 1$.

3. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle -2x, z, y + 1\rangle$ and S is the part of the plane given by $\mathbf{r}(u, v) = \langle u + v, 2 - 3v, 5 + 2u + v\rangle$, $0 \le u \le 3$, $0 \le v \le 2$, with upward orientation.

4. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle xy, 8x^2, yz\rangle$ and S is the part of the surface $z = xe^y$, $0 \le x \le 2$, $-1 \le y \le 1$, with upward orientation.

5. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle x + y^2,\ y + z^2,\ x^2 + z\rangle$ and C is the triangle with vertices $(3, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 3)$, oriented counterclockwise when viewed from above.

6. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle yz,\ 9xz,\ e^{xy}\rangle$ and C is the circle $x^2 + y^2 = 4$, $z = 1$, oriented counterclockwise when viewed from above.

7. Use Stokes' Theorem to express $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ as a single integral (do not evaluate), where $\mathbf{F}(x, y, z) = y\cos(z)\,\mathbf{i} + e^x \sin(z)\,\mathbf{j} + xe^y\,\mathbf{k}$ and S is the hemisphere $z = \sqrt{49 - x^2 - y^2}$, oriented upward.

8. Let $\mathbf{F}(x, y, z) = \langle y^4 - e^z,\ x^2y + 7,\ \cos(xy) + y^2z\rangle$ and S be the surface of the solid bounded by the paraboloid $z = 2x^2 + 2y^2$ and $z = 8$. Use the Divergence Theorem to find the flux of $\mathbf{F}$ across S.

9. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle x^3 + y,\ y^3,\ z^3 + e^x\rangle$ and S is the surface of the solid bounded by the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and the xy-plane.

Solutions

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 10 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Evaluate $\iint_S (z + xy)\, dS$, where S is the part of the cylinder $x^2 + y^2 = 4$ that lies between the planes $z = 0$ and $z = 4$.

   $\boxed{32\pi}$

2. Evaluate $\iint_S xz\, dS$, where S is the surface $y = 2x + z^2$, $2 \le x \le 4$, $0 \le z \le 1$.

   $\boxed{\frac{1}{2}\left(27 - 5^{3/2}\right)}$

3. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle -2x, z, y + 1\rangle$ and S is the part of the plane given by $\mathbf{r}(u, v) = \langle u + v, 2 - 3v, 5 + 2u + v\rangle$, $0 \le u \le 3$, $0 \le v \le 2$, with upward orientation.

   $\boxed{126}$

4. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle xy, 8x^2, yz\rangle$ and S is the part of the surface $z = xe^y$, $0 \le x \le 2$, $-1 \le y \le 1$, with upward orientation.

   $\boxed{-32(e - e^{-1})}$

5. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle x + y^2,\ y + z^2,\ x^2 + z\rangle$ and C is the triangle with vertices $(3, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 3)$, oriented counterclockwise when viewed from above.

   $\boxed{-27}$

6. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle yz,\ 9xz,\ e^{xy}\rangle$ and C is the circle $x^2 + y^2 = 4$, $z = 1$, oriented counterclockwise when viewed from above.

   $\boxed{32\pi}$

7. Use Stokes' Theorem to express $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$ as a single integral (do not evaluate), where $\mathbf{F}(x, y, z) = y\cos(z)\,\mathbf{i} + e^x \sin(z)\,\mathbf{j} + xe^y\,\mathbf{k}$ and S is the hemisphere $z = \sqrt{49 - x^2 - y^2}$, oriented upward.

   $\boxed{\int_0^{2\pi} -49\sin^2 t\, dt}$

8. Let $\mathbf{F}(x,y,z) = \langle y^4 - e^z,\ x^2y + 7,\ \cos(xy) + y^2z \rangle$ and S be the surface of the solid bounded by the paraboloid $z = 2x^2 + 2y^2$ and $z = 8$. Use the Divergence Theorem to find the flux of $\mathbf{F}$ across S.

   $\boxed{\dfrac{64\pi}{3}}$

9. Use the Divergence Theorem to evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x,y,z) = \langle x^3 + y,\ y^3,\ z^3 + e^x \rangle$ and S is the surface of the solid bounded by the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and the xy-plane.

   $\boxed{\dfrac{6\pi}{5}}$