



MATH 251 - WEEK-IN-REVIEW 2

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PROBLEM STATEMENTS

You should attempt the problems yourself first. The next section contains the solutions.

- Find an equation of the plane passing through the point $(1, 5, -3)$ and perpendicular to the line $x = 2 - 4t$, $y = 2t$, $z = -1 + t$.
- Consider the lines $\mathbf{r}_1(t) = \langle 2 + t, 2t, 5 + t \rangle$ and $\mathbf{r}_2(s) = \langle s, -4 + 4s, 3 + s \rangle$.
 - Find the point at which the given lines intersect.
 - Find an equation of the plane that contains these lines.
- Describe and sketch the following surfaces.
 - $-4x^2 + y^2 = 4 + 2z^2$
 - $16z^2 - 4y^2 = 9x^2$
- Find the domain of $\mathbf{r}(t) = \left\langle \frac{7}{t^2 - 1}, \ln(t + 5), \sqrt{2 - t} \right\rangle$.
- Find $\lim_{t \rightarrow 2} \left\langle t^2, \frac{2 - t}{t^2 - 4}, \frac{\sin(\pi t)}{e^{t-2} - 1} \right\rangle$.
- Find a vector equation for the tangent line to the curve $\mathbf{r}(t) = \left\langle 4\sqrt{t}, t^2 - 10, \frac{4}{t} \right\rangle$ at $(8, 6, 1)$.
- The curves $\mathbf{r}_1(s) = \langle s, s^2, 2e^{3s-3} \rangle$ and $\mathbf{r}_2(t) = \langle \cos(t), t^3 + t + 1, 3t + 2 \rangle$ intersect at $(1, 1, 2)$. Find their angle of intersection.
- Find the length of the curve $\mathbf{r}(t) = \langle 3 \cos(t), 3 \sin(t), 2t \rangle$ from $(3, 0, 0)$ to $(-3, 0, 2\pi)$.
- Find the curvature of $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $(1, 1, 1)$.
- The velocity of an object is given by $\mathbf{v}(t) = \langle 4t \ln t, \cos(2t - 2), 4t^3 \rangle$. Find the position $\mathbf{r}(t)$ if $\mathbf{r}(1) = \langle 1, -3, 0 \rangle$.



SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. You can also see them all by viewing the [Week 2 playlist \(clickable link\)](#). You can turn on closed captions by clicking “CC” inside YouTube as well as adjust the video speed inside of “Settings” by clicking the cog in the bottom right of the player.

1. Find an equation of the plane passing through the point $(1, 5, -3)$ and perpendicular to the line $x = 2 - 4t, y = 2t, z = -1 + t$.

$$-4x + 2y + z = 3 \text{ or some scalar multiple of this}$$

2. Consider the lines $\mathbf{r}_1(t) = \langle 2 + t, 2t, 5 + t \rangle$ and $\mathbf{r}_2(s) = \langle s, -4 + 4s, 3 + s \rangle$.

- (a) Find the point at which the given lines intersect.

$$(0, -4, 3)$$

- (b) Find an equation of the plane that contains these lines.

$$-2x + 2z = 6 \text{ or some scalar multiple of this}$$

3. Describe and sketch the following surfaces.

(a) $-4x^2 + y^2 = 4 + 2z^2$

Hyperboloid of 2 sheets that opens up along the y -axis (see video for sketch)

(b) $16z^2 - 4y^2 = 9x^2$

Elliptic cone that opens up along z -axis (see video for sketch)

4. Find the domain of $\mathbf{r}(t) = \left\langle \frac{7}{t^2 - 1}, \ln(t + 5), \sqrt{2 - t} \right\rangle$.

$$(-5, -1) \cup (-1, 1) \cup (1, 2]$$

5. Find $\lim_{t \rightarrow 2} \left\langle t^2, \frac{2 - t}{t^2 - 4}, \frac{\sin(\pi t)}{e^{t-2} - 1} \right\rangle$.

$$\left\langle 4, -\frac{1}{4}, \pi \right\rangle$$

6. Find a vector equation for the tangent line to the curve $\mathbf{r}(t) = \left\langle 4\sqrt{t}, t^2 - 10, \frac{4}{t} \right\rangle$ at $(8, 6, 1)$.

$$\mathbf{r}(t) = \left\langle 8 + t, 6 + 8t, 1 - \frac{1}{4}t \right\rangle$$



7. The curves $\mathbf{r}_1(s) = \langle s, s^2, 2e^{3s-3} \rangle$ and $\mathbf{r}_2(t) = \langle \cos(t), t^3 + t + 1, 3t + 2 \rangle$ intersect at $(1, 1, 2)$. Find their angle of intersection.

$$\theta = \cos^{-1} \left(\frac{20}{\sqrt{410}} \right)$$

8. Find the length of the curve $\mathbf{r}(t) = \langle 3 \cos(t), 3 \sin(t), 2t \rangle$ from $(3, 0, 0)$ to $(-3, 0, 2\pi)$.

$$\sqrt{13}\pi$$

9. Find the curvature of $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $(1, 1, 1)$.

$$\frac{\sqrt{76}}{14^{3/2}}$$

10. The velocity of an object is given by $\mathbf{v}(t) = \langle 4t \ln t, \cos(2t - 2), 4t^3 \rangle$. Find the position $\mathbf{r}(t)$ if $\mathbf{r}(1) = \langle 1, -3, 0 \rangle$.

$$\mathbf{r}(t) = \left\langle 2t^2 \ln t - t^2 + 2, \frac{1}{2} \sin(2t - 2) - 3, t^4 - 1 \right\rangle$$