



MATH 251 - WEEK-IN-REVIEW 3

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PROBLEM STATEMENTS

You should attempt the problems yourself first. The next section contains the solutions.

1. Find and sketch the domain of the following functions:

(a) $f(x, y) = \ln(x^2 + y^2 - 1) + \sqrt{9 - x^2 - y^2}$

(b) $f(x, y) = \frac{1}{\sqrt[4]{3xy}}$

2. Sketch the graphs of the following functions:

(a) $f(x, y) = 6 - 2x - 3y$

(b) $f(x, y) = 2 - x^2 - 3y^2$

3. Sketch several level curves for the following functions:

(a) $f(x, y) = \sqrt{(x-2)^2 + y^2}$

(b) $f(x, y) = x - y^2$

4. Describe the level surfaces of $f(x, y, z) = 3x^2 + 3y^2 + 3z^2$.

5. Find f_x and f_y for $f(x, y) = \ln(3xy + x^2y^3) + 2y^2$.

6. Find f_u and f_v for $f(u, v) = v^3 e^{\sin(5u-4v)}$.

7. Find f_x , f_y , and f_z for $f(x, y, z) = \frac{y^2 \sec(z)}{x} + 2x\sqrt[3]{z^4}$.

8. Find all second-order partial derivatives for $f(x, y) = \cos(5x) - x^2e^{2y} + 4y^3$.

9. Find an equation of the tangent plane to the surface $z = xy^2 - 2x^3$ at the point $(1, -2, 2)$.

10. Use the linearization of $f(x, y) = 2e^{x^3-y^2}$ at $(1, -1)$ to approximate $f(1.1, -1.1)$.

11. Find the differential of $z = x \tan(xy + 1)$.

12. The height and radius of a cone are measured to be $h = 4$ cm and $r = 3$ cm with a maximum error in measurement of 0.1 cm in both. Use differentials to estimate the maximum error in calculating the volume of the cone.



SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. You can also see them all by viewing the [Week 3 playlist \(clickable link\)](#). You can turn on closed captions by clicking “CC” inside YouTube as well as adjust the video speed inside of “Settings” by clicking the cog in the bottom right of the player.

1. Find and sketch the domain of the following functions:

(a) $f(x, y) = \ln(x^2 + y^2 - 1) + \sqrt{9 - x^2 - y^2}$

All (x, y) such that $1 < x^2 + y^2 \leq 9$ (see video for sketch)

(b) $f(x, y) = \frac{1}{\sqrt[3]{3xy}}$

All (x, y) such that $xy > 0$ (see video for sketch)

2. Sketch the graphs of the following functions:

(a) $f(x, y) = 6 - 2x - 3y$

The graph of f is a plane (see video for sketch).

(b) $f(x, y) = 2 - x^2 - 3y^2$

The graph of f is an elliptic paraboloid (see video for sketch).

3. Sketch several level curves for the following functions:

(a) $f(x, y) = \sqrt{(x - 2)^2 + y^2}$

The level curves are circles with center $(2, 0)$ (see video for sketch).

(b) $f(x, y) = x - y^2$

The level curves are parabolas opening to the right (see video for sketch).

4. Describe the level surfaces of $f(x, y, z) = 3x^2 + 3y^2 + 3z^2$

A family of spheres with center $(0, 0, 0)$.

5. Find f_x and f_y for $f(x, y) = \ln(3xy + x^2y^3) + 2y^2$.

$$f_x = \frac{3y + 2xy^3}{3xy + x^2y^3}, \quad f_y = \frac{3x + 3x^2y^2}{3xy + x^2y^3} + 4y$$

6. Find f_u and f_v for $f(u, v) = v^3 e^{\sin(5u-4v)}$.

$$f_u = 5v^3 e^{\sin(5u-4v)} \cos(5u - 4v), \quad f_v = -4v^3 e^{\sin(5u-4v)} \cos(5u - 4v) + 3v^2 e^{\sin(5u-4v)}$$



7. Find f_x , f_y , and f_z for $f(x, y, z) = \frac{y^2 \sec(z)}{x} + 2x\sqrt[3]{z^4}$.

$$f_x = -x^{-2}y \sec(z) + 2z^{4/3}, \quad f_y = 2x^{-1}y \sec(z), \quad f_z = xy^2 \sec(z) \tan(z) + \frac{8}{3}xz^{1/3}$$

8. Find all second-order partial derivatives for $f(x, y) = \cos(5x) - x^2e^{2y} + 4y^3$

$$f_{xx} = -25 \cos(5x) - 2e^{2y}, \quad f_{xy} = -4xe^{2y}, \quad f_{yx} = -4xe^{2y}, \quad f_{yy} = -4x^2e^{2y} + 24y$$

9. Find an equation of the tangent plane to the surface $z = xy^2 - 2x^3$ at the point $(1, -2, 2)$.

$$2x + 4y + z = -4 \text{ (or any scalar multiple of this)}$$

10. Use the linearization of $f(x, y) = 2e^{x^3 - y^2}$ at $(1, -1)$ to approximate $f(1.1, -1.1)$.

$$f(1.1, -1.1) \approx 2.2$$

11. Find the differential of $z = x \tan(xy + 1)$.

$$dz = [xy \sec^2(xy + 1) + \tan(xy + 1)] dx + x^2 \sec^2(xy + 1) dy$$

12. The height and radius of a cone are measured to be $h = 4$ cm and $r = 3$ cm with a maximum error in measurement of 0.1 cm in both. Use differentials to estimate the maximum error in calculating the volume of the cone.

$$dV = 1.1\pi \text{ cm}^3$$