Math 251 - Week-In-Review 4

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Sand is being poured onto a cone-shaped pile. The height of the pile is increasing at a rate of 6 cm/min and the radius is increasing at a rate of 3 cm/min. At what rate is the volume of the sand pile changing when the radius is 2 cm and the height is 20 cm?

2. Let $w = tue^{2v}$, where $t = x^2\sqrt{y}$ and $u = 3x - 4y$ and $v = 2x\ln(y)$. Find $\dfrac{\partial w}{\partial x}$ and $\dfrac{\partial w}{\partial y}$. What is $\dfrac{\partial w}{\partial y}$ when $x = 2$, $y = 1$?

3. Find the directional derivative of $f(x, y) = x^2e^{xy}$ at $P(1, 0)$ in the direction from P to $Q(5, 3)$.

4. Find the rate of change of $f(x, y, z) = xy^2z^3 - 2xz + 4$ at $(2, 1, 1)$ in the direction of $\langle -2, 9, 6\rangle$.

5. Find the maximum rate of change of $f(x, y) = x^2\cos(xy)+3y$ at $(3, 0)$ and the direction in which it occurs.

6. Consider the surface defined by $3xy - 2xz + yz = 3$.
   (a) Find an equation of the tangent plane to the surface at $(1, -1, -2)$.
   (b) Find parametric equations for the normal line to the surface at $(1, -1, -2)$.

7. Find the critical points of $f(x, y) = 2x^3 + xy^2 + 6x^2 + y^2 + 1$.

8. Find the locations of local extrema and saddle points of $f(x, y) = 3x^2 + \frac{1}{3}y^3 + y^2 - 8y$. Classify any local extrema as maxima or minima.

9. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x^2y + 4$ on the rectangle $D = \{(x, y) \mid -1 \le x \le 1, -1 \le y \le 1\}$.

10. Find the absolute extrema of $f(x, y) = 8 + xy - x - 2y$ on the closed triangular region D with vertices $(1, 0)$, $(5, 0)$, and $(1, 4)$.

11. Use the method of Lagrange multipliers to find the extreme values of $f(x, y) = y^2 - x^2$ subject to the constraint $\dfrac{1}{4}x^2 + y^2 = 25$.

Solutions

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 4 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Sand is being poured onto a cone-shaped pile. The height of the pile is increasing at a rate of 6 cm/min and the radius is increasing at a rate of 3 cm/min. At what rate is the volume of the sand pile changing when the radius is 2 cm and the height is 20 cm?

   $\boxed{88\pi}$

2. Let $w = tue^{2v}$, where $t = x^2\sqrt{y}$ and $u = 3x - 4y$ and $v = 2x\ln(y)$. Find $\dfrac{\partial w}{\partial x}$ and $\dfrac{\partial w}{\partial y}$. What is $\dfrac{\partial w}{\partial y}$ when $x = 2$, $y = 1$?

   $\boxed{52}$

3. Find the directional derivative of $f(x, y) = x^2e^{xy}$ at $P(1, 0)$ in the direction from P to $Q(5, 3)$.

   $\boxed{\dfrac{11}{5}}$

4. Find the rate of change of $f(x, y, z) = xy^2z^3 - 2xz + 4$ at $(2, 1, 1)$ in the direction of $\langle -2, 9, 6\rangle$.

   $\boxed{\dfrac{50}{11}}$

5. Find the maximum rate of change of $f(x, y) = x^2\cos(xy) + 3y$ at $(3, 0)$ and the direction in which it occurs.

   $\boxed{\text{direction: } \langle 6, 3\rangle, \quad \text{maximum rate of change: } \sqrt{45}}$

6. Consider the surface defined by $3xy - 2xz + yz = 3$.

   (a) Find an equation of the tangent plane to the surface at $(1, -1, -2)$.

   (b) Find parametric equations for the normal line to the surface at $(1, -1, -2)$.

   $\boxed{x + y - 3z = 6; \quad x = 1 + t,\ y = -1 + t,\ z = -2 - 3t}$

7. Find the critical points of $f(x, y) = 2x^3 + xy^2 + 6x^2 + y^2 + 1$.

   $\boxed{(0, 0),\ (-2, 0),\ \left(-1, -\sqrt{6}\right),\ \left(-1, \sqrt{6}\right)}$

8. Find the locations of local extrema and saddle points of $f(x, y) = 3x^2 + \frac{1}{3}y^3 + y^2 - 8y$. Classify any local extrema as maxima or minima.

   $\boxed{\text{saddle point at } (0, -4), \text{ local minimum at } (0, 2)}$

9. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x^2y + 4$ on the rectangle $D = \{(x, y) \mid -1 \leq x \leq 1,\ -1 \leq y \leq 1\}$.

   absolute minimum = 4; absolute maximum = 7

10. Find the absolute extrema of $f(x, y) = 8 + xy - x - 2y$ on the closed triangular region D with vertices $(1, 0)$, $(5, 0)$, and $(1, 4)$.

    absolute minimum = 3, absolute maximum = 7

11. Use the method of Lagrange multipliers to find the extreme values of $f(x, y) = y^2 - x^2$ subject to the constraint $\frac{1}{4}x^2 + y^2 = 25$.

    absolute minimum = −100, absolute maximum = 25

    Video errata: At 4:10 I write $y = \pm 25$, when I should have written $y = \pm 5$. I correct myself near the end of the video.