Math 251 - Week-In-Review 5

Justin Cantu

Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Evaluate $\int_0^1 \int_1^2 (4x + 3y^2 - x^2y)\, dy\, dx$.

2. Find $\iint_R y^2 \sec^2(x)\, dA$, where $R = \left\{(x, y) \mid 0 \leq x \leq \frac{\pi}{4},\, 1 \leq y \leq 3\right\}$.

3. Find $\iint_R x\sqrt{y + 4}\, dA$, where $R = [1, 2] \times [-4, 0]$.

4. Find $\iint_R y \cos(xy)\, dA$, where $R = [0, 1] \times [0, \pi]$.

5. Find the volume of the solid that lies under the plane $x + 2y + z = 6$ and above the rectangle $R = [0, 2] \times [0, 1]$.

6. Sketch the region of integration and evaluate $\iint_D (x^2 - 4xy)\, dA$, where D is the region bounded by the lines $y = x$, $y = 0$, $x = 1$, and $x = 2$.

7. Sketch the region of integration and evaluate $\iint_D xe^y\, dA$, where D is the region bounded by $y = 0$, $y = x^2$, and $x = 3$.

8. Sketch the region of integration and evaluate $\int_0^2 \int_y^{2y} xy^2\, dy\, dx$.

9. Change the order of integration for $\int_0^2 \int_{y^2}^4 f(x, y)\, dx\, dy$.

10. Evaluate $\int_0^1 \int_x^1 e^{-y^2}\, dy\, dx$ by reversing the order of integration.

11. Set up but do not evaluate Type I and Type II integrals that give the volume of the solid under the surface $z = 2xy$ and above the triangle with vertices $(1, 1)$, $(1, 2)$, and $(2, 1)$.

SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 5 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Evaluate $\int_0^1 \int_1^2 (4x + 3y^2 - x^2y)\, dy\, dx$.

   $\boxed{\dfrac{17}{2}}$

2. Find $\iint_R y^2 \sec^2(x)\, dA$, where $R = \left\{(x, y) \mid 0 \le x \le \frac{\pi}{4},\ 1 \le y \le 3\right\}$.

   $\boxed{\dfrac{26}{3}}$

3. Find $\iint_R x\sqrt{y + 4}\, dA$, where $R = [1, 2] \times [-4, 0]$.

   $\boxed{8}$

4. Find $\iint_R y\cos(xy)\, dA$, where $R = [0, 1] \times [0, \pi]$.

   $\boxed{2}$

5. Find the volume of the solid that lies under the plane $x + 2y + z = 6$ and above the rectangle $R = [0, 2] \times [0, 1]$.

   $\boxed{8}$

6. Sketch the region of integration and evaluate $\iint_D (x^2 - 4xy)\, dA$, where D is the region bounded by the lines $y = x$, $y = 0$, $x = 1$, and $x = 2$.

   $\boxed{-\dfrac{15}{4} \text{ (see video for sketch)}}$

7. Sketch the region of integration and evaluate $\iint_D xe^y\, dA$, where D is the region bounded by $y = 0$, $y = x^2$, and $x = 3$.

   $\boxed{\dfrac{1}{2}e^9 - 5 \text{ (see video for sketch)}}$

8. Sketch the region of integration and evaluate $\displaystyle\int_0^2 \int_y^{2y} xy^2\, dy\, dx$.

$\dfrac{48}{5}$ (see video for sketch)

9. Change the order of integration for $\displaystyle\int_0^2 \int_{y^2}^{4} f(x,y)\, dx\, dy$.

$\displaystyle\int_0^4 \int_0^{\sqrt{x}} f(x,y)\, dy\, dx$

10. Evaluate $\displaystyle\int_0^1 \int_x^1 e^{-y^2}\, dy\, dx$ by reversing the order of integration.

$-\dfrac{1}{2}e^{-1} + \dfrac{1}{2}$

11. Set up but do not evaluate Type I and Type II integrals that give the volume of the solid under the surface $z = 2xy$ and above the triangle with vertices $(1,1)$, $(1,2)$, and $(2,1)$.

Type I: $V = \displaystyle\int_1^2 \int_1^{3-x} 2xy\, dy\, dx$ Type II: $V = \displaystyle\int_1^2 \int_1^{3-y} 2xy\, dx\, dy$