Math 251 - Week-In-Review 6

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Evaluate $\iint_R (x+4)\,dA$, where $R = \{(x,y) \mid x^2+y^2 \leq 9,\, x \geq 0\}$.

2. Evaluate $\iint_R xy\,dA$, where R is the region in the first quadrant enclosed by the circle $x^2+y^2=4$ and the lines $x=0$ and $y=x$.

3. Evaluate $\iint_R 6y\,dA$, where R is the region in the second quadrant between the circles $x^2+y^2=1$ and $x^2+y^2=9$.

4. Convert $\int_{-2}^{0}\int_{0}^{\sqrt{-x^2-2x}} (x^2+y^2+1)\,dy\,dx$ to polar coordinates.

5. Set up an iterated integral in polar coordinates that gives the volume of the solid that lies above the xy-plane, below the paraboloid $z = 9-x^2-y^2$, and inside the cylinder $x^2+y^2=4$.

6. Set up an iterated integral in polar coordinates that gives the volume of the solid bounded by the cone $z=\sqrt{x^2+y^2}$ and the plane $z=4$.

7. Express $\iiint_E xy^4z\,dV$ as an iterated integral in the order $dy\,dz\,dx$, where E is the tetrahedron bounded by the coordinate planes and $3x+4y+3z=6$.

8. Evaluate $\iiint_E 6xy\,dV$, where E is the solid bounded by $y=x^2$, $x=y^2$, $z=0$, and $z=x+2y$.

9. Set up an iterated integral that gives the volume of the solid enclosed by the cylinder $x^2+y^2=9$ and the planes $y+z=8$ and $z=2$.

Solutions

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 6 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Evaluate $\displaystyle\iint_R (x+4)\,dA$, where $R = \{(x,y) \mid x^2+y^2 \leq 9,\ x \geq 0\}$.

   $\boxed{18+18\pi}$

2. Evaluate $\displaystyle\iint_R xy\,dA$, where R is the region in the first quadrant enclosed by the circle $x^2+y^2=4$ and the lines $x=0$ and $y=x$.

   $\boxed{1}$

3. Evaluate $\displaystyle\iint_R 6y\,dA$, where R is the region in the second quadrant between the circles $x^2+y^2=1$ and $x^2+y^2=9$.

   $\boxed{52}$

4. Convert $\displaystyle\int_{-2}^{0}\int_{0}^{\sqrt{-x^2-2x}} (x^2+y^2+1)\,dy\,dx$ to polar coordinates.

   $$\boxed{\int_{\pi/2}^{\pi}\int_{0}^{-2\cos\theta} (r^3+r)\,dr\,d\theta}$$

5. Set up an iterated integral in polar coordinates that gives the volume of the solid that lies above the xy-plane, below the paraboloid $z=9-x^2-y^2$, and inside the cylinder $x^2+y^2=4$.

   $$\boxed{\int_{0}^{2\pi}\int_{0}^{2} (9r-r^3)\,dr\,d\theta}$$

   Video note: I forgot to mention that the paraboloid $z=9-x^2-y^2$ intersects the xy-plane in the circle of radius 3 described by $x^2+y^2=9$. Thus a portion of the cylinder $x^2+y^2=4$ is strictly inside the paraboloid.

6. Set up an iterated integral in polar coordinates that gives the volume of the solid bounded by the cone $z=\sqrt{x^2+y^2}$ and the plane $z=4$.

   $$\boxed{\int_{0}^{2\pi}\int_{0}^{4} (4r-r^2)\,dr\,d\theta}$$

7. Express $\iiint_E xy^4z\,dV$ as an iterated integral in the order $dy\,dz\,dx$, where E is the tetrahedron bounded by the coordinate planes and $3x + 4y + 3z = 6$.

$$\int_0^2 \int_0^{2-x} \int_0^{\frac{3}{2}-\frac{3}{4}x-\frac{3}{4}z} xy^4z\,dy\,dz\,dx$$

8. Evaluate $\iiint_E 6xy\,dV$, where E is the solid bounded by $y = x^2$, $x = y^2$, $z = 0$, and $z = x + 2y$.

$$\frac{27}{28}$$

9. Set up an iterated integral that gives the volume of the solid enclosed by the cylinder $x^2+y^2 = 9$ and the planes $y + z = 8$ and $z = 2$.

$$\int_0^{2\pi} \int_0^3 \int_2^{8-r\sin\theta} r\,dz\,dr\,d\theta \quad \text{or} \quad \int_0^{2\pi} \int_0^3 (6r - r^2\sin\theta)\,dr\,d\theta$$

Video note: I set up the volume by starting with a triple integral, but you can also use a double integral.