Math 251 - Week-In-Review 7

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Evaluate $\iiint_E \sqrt{x^2+y^2}\,dV$, where E is the solid between the elliptic paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$.

2. Express $\iiint_E x\,dV$ as an iterated integral in cylindrical coordinates, where E is the solid that lies inside the cylinder $x^2 + y^2 = 4$ and inside the sphere $x^2 + y^2 + z^2 = 16$.

3. Convert $\int_{-3}^{3}\int_{0}^{\sqrt{9-y^2}}\int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy$ to cylindrical and spherical coordinates.

4. Using spherical coordinates, find the volume of the part of the ball $\rho \le 3$ that lies between the cones $\phi = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$.

5. Evaluate $\iiint_E z^2\,dV$, where E is bounded by the xy-plane and the hemispheres $z = \sqrt{1-x^2-y^2}$ and $z = \sqrt{9-x^2-y^2}$.

6. Use the transformation $u = 3x$, $v = 2y$ to evaluate $\iint_R e^{9x^2+4y^2}\,dA$, where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.

7. Use the transformation $x = 2u + v$, $y = u + 2v$ to express $\iint_R (3x - y)\,dA$ as an iterated integral in the order $dv\,du$, where R is the triangular region with vertices $(0, 0)$, $(2, 1)$, and $(1, 2)$.

SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 7 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Evaluate $\iiint_E \sqrt{x^2+y^2}\, dV$, where E is the solid between the elliptic paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$.

   $\boxed{\dfrac{648\pi}{3}}$

2. Express $\iiint_E x\, dV$ as an iterated integral in cylindrical coordinates, where E is the solid that lies inside the cylinder $x^2 + y^2 = 4$ and inside the sphere $x^2 + y^2 + z^2 = 16$.

   $\boxed{\int_0^{2\pi} \int_0^2 \int_{-\sqrt{16-r^2}}^{\sqrt{16-r^2}} r^2 \cos\theta\, dz\, dr\, d\theta}$

3. Convert $\int_{-3}^{3} \int_0^{\sqrt{9-y^2}} \int_0^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\, dz\, dx\, dy$ to cylindrical and spherical coordinates.

   $\boxed{\text{Cyl: } \int_{-\pi/2}^{\pi/2} \int_0^3 \int_0^{\sqrt{9-r^2}} rz\sqrt{r^2+z^2}\, dz\, dr\, d\theta, \qquad \text{Sph: } \int_0^{\pi/2} \int_{-\pi/2}^{\pi/2} \int_0^3 \rho^4 \cos\phi \sin\phi\, d\rho\, d\theta\, d\phi}$

4. Using spherical coordinates, find the volume of the part of the ball $\rho \le 3$ that lies between the cones $\phi = \dfrac{\pi}{6}$ and $\phi = \dfrac{\pi}{3}$.

   $\boxed{9\pi\left(\sqrt{3}-1\right)}$

5. Evaluate $\iiint_E z^2\, dV$, where E is bounded by the xy-plane and the hemispheres $z = \sqrt{1-x^2-y^2}$ and $z = \sqrt{9-x^2-y^2}$.

   $\boxed{\dfrac{484\pi}{15}}$

6. Use the transformation $u = 3x$, $v = 2y$ to evaluate $\iint_R e^{9x^2+4y^2}\, dA$, where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.

   $\boxed{\dfrac{\pi}{24}(e-1)}$

7. Use the transformation $x = 2u + v$, $y = u + 2v$ to express $\iint_R (3x - y)\, dA$ as an iterated integral in the order $dv\, du$, where R is the triangular region with vertices $(0, 0)$, $(2, 1)$, and $(1, 2)$.

$$\int_0^1 \int_0^{1-u} (15u + 3v)\, dv\, du$$