Math 251 - Week-In-Review 8

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Evaluate $\int_C xy^2\,ds$, where C is the right half of the circle $x^2+y^2=9$, oriented counterclockwise.

2. Evaluate $\int_C xyz\,ds$, where C is the curve given by $\mathbf{r}(t) = \langle 2t, t+1, 2-t\rangle$, $-1 \le t \le 0$.

3. Evaluate $\int_C 2y\,ds$, where C is the arc of the curve $x = y^2$ from $(1,-1)$ to $(4,2)$.

4. Evaluate $\int_C y\,dx + x^2\,dy$, where C is the curve given by $\mathbf{r}(t) = \langle 3e^t, e^{2t}\rangle$, $0 \le t \le 1$.

5. Evaluate $\int_C xy\,dx + x^2\,dy + z\,dz$ where C is the line segment from $(0,-1,1)$ to $(2,3,-1)$.

6. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle y^2, \sin(x-1)\rangle$ and C is the line segment from $(1,4)$ to $(3,-1)$.

7. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z) = \langle -x, z^2, 2y\rangle$ and C is the curve given by $\mathbf{r}(t) = \langle t, t^3, 2t\rangle$, $0 \le t \le 1$.

8. Find the work done by the force field $\mathbf{F}(x,y) = \langle x^2+y, 2y\rangle$ in moving a particle along the arc of $y = x^2$ from $(-1,1)$ to $(2,4)$.

9. Is $\mathbf{F}(x,y) = \langle xy+x, x^2+4y\rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.

10. Is $\mathbf{F}(x,y) = \langle 2x+4y, 4x-1\rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.

11. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle 4x^3y, x^4\rangle$ and C is the curve $\mathbf{r}(t) = \langle t^4-4t^2+1, t^3+3\rangle$, $0 \le t \le 2$.

12. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle 1-y, y-x\rangle$ and C is *any* path from $(1,-1)$ to $(3,2)$.

13. Given that $\mathbf{F}(x,y,z) = \langle 6xy, 3x^2+z, y-6z^2\rangle$ is conservative, evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the curve $x = t+1$, $y = 2t^2$, $z = 3t-1$, $0 \le t \le 1$.

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SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 8 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Evaluate $\int_C xy^2\, ds$, where C is the right half of the circle $x^2+y^2=9$, oriented counterclockwise.

   $\boxed{54}$

2. Evaluate $\int_C xyz\, ds$, where C is the curve given by $\mathbf{r}(t)=\langle 2t, t+1, 2-t\rangle$, $-1\le t\le 0$.

   $\boxed{-\frac{7\sqrt{6}}{6}}$

3. Evaluate $\int_C 2y\, ds$, where C is the arc of the curve $x=y^2$ from $(1,-1)$ to $(4,2)$.

   $\boxed{\frac{1}{6}\left(17^{3/2}-5^{3/2}\right)}$

4. Evaluate $\int_C y\, dx + x^2\, dy$, where C is the curve given by $\mathbf{r}(t)=\langle 3e^t, e^{2t}\rangle$, $0\le t\le 1$.

   $\boxed{e^3+\frac{9}{2}e^4-\frac{11}{2}}$

5. Evaluate $\int_C xy\, dx + x^2\, dy + z\, dz$ where C is the line segment from $(0,-1,1)$ to $(2,3,-1)$.

   $\boxed{\frac{26}{3}}$

6. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y)=\langle y^2, \sin(x-1)\rangle$ and C is the line segment from $(1,4)$ to $(3,-1)$.

   $\boxed{\frac{1}{2}+\frac{5}{2}\cos(2)}$

7. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z)=\langle -x, z^2, 2y\rangle$ and C is the curve given by $\mathbf{r}(t)=\langle t, t^3, 2t\rangle$, $0\le t\le 1$.

   $\boxed{\frac{29}{10}}$

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8. Find the work done by the force field $\mathbf{F}(x, y) = \langle x^2 + y, 2y \rangle$ in moving a particle along the arc of $y = x^2$ from $(-1, 1)$ to $(2, 4)$.

   $\boxed{21}$

9. Is $\mathbf{F}(x, y) = \langle xy + x, x^2 + 4y \rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.

   $\boxed{\text{No.}}$

10. Is $\mathbf{F}(x, y) = \langle 2x + 4y, 4x - 1 \rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.

    $\boxed{\text{Yes, } f(x, y) = 4xy + x^2 - y \text{ (you can also have } +C \text{ for some nonzero constant } C\text{).}}$

11. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle 4x^3 y, x^4 \rangle$ and C is the curve $\mathbf{r}(t) = \langle t^4 - 4t^2 + 1, t^3 + 3 \rangle$, $0 \leq t \leq 2$.

    $\boxed{8}$

12. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle 1 - y, y - x \rangle$ and C is *any* path from $(1, -1)$ to $(3, 2)$.

    $\boxed{-\frac{7}{2}}$

13. Given that $\mathbf{F}(x, y, z) = \langle 6xy, 3x^2 + z, y - 6z^2 \rangle$ is conservative, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $x = t + 1$, $y = 2t^2$, $z = 3t - 1$, $0 \leq t \leq 1$.

    $\boxed{10}$

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