Math 251 - Week-In-Review 9

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Evaluate $\oint_C y^2\,dx + 3x\,dy$, where C is the triangle with vertices $(1,1)$, $(3,1)$, and $(2,2)$, oriented counterclockwise.

2. Evaluate $\oint_C (\cos(e^x) + y^2)\,dx + (x^2 + \sqrt{y^3+y})\,dy$, where C consists of the line segment from $(-3,0)$ to $(3,0)$ and the top half of $x^2 + y^2 = 9$. Assume counterclockwise orientation.

3. Evaluate $\oint_C (y^2 + x^{10})\,dx + y^7\,dy$, where C is the curve that encloses the region bounded by $x = 0$, $y = 0$, and $y = x^2 - 1$, traversed counterclockwise.

4. Suppose a particle travels one revolution clockwise around the circle $x^2 + y^2 = 4$ under the force field $\mathbf{F}(x,y) = \langle y^3 + \sin(x), e^y - x^3 \rangle$. Find the work done by $\mathbf{F}$.

5. Find the divergence and curl of $\mathbf{F}(x,y,z) = xe^{yz}\,\mathbf{j} + x^3z\,\mathbf{k}$.

6. Find the divergence and curl of $\mathbf{F}(x,y,z) = \langle xyz, 2\sin(xz), y^2z^3 \rangle$.

7. Is $\mathbf{F}(x,y,z) = \langle 3x^2y + 4y^2z, x^3 + 8xyz, 4xy^2 - e^z \rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.

8. Find the area of the part of the plane given by $\mathbf{r}(u,v) = \langle 2+v, u-3v, 9-2u+v \rangle$, $-1 \le u \le 1$, $0 \le v \le 3$.

9. Find the area of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 49$.

10. Find the area of the part of the plane $2x + 5y + z = 10$ that lies in the first octant.

Solutions

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 9 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Evaluate $\oint_C y^2\,dx + 3x\,dy$, where C is the triangle with vertices $(1,1)$, $(3,1)$, and $(2,2)$, oriented counterclockwise.

   $\boxed{\dfrac{1}{3}}$

2. Evaluate $\oint_C (\cos(e^x) + y^2)\,dx + (x^2 + \sqrt{y^3+y})\,dy$, where C consists of the line segment from $(-3,0)$ to $(3,0)$ and the top half of $x^2+y^2=9$. Assume counterclockwise orientation.

   $\boxed{-36}$

3. Evaluate $\oint_C (y^2+x^{10})\,dx + y^7\,dy$, where C is the curve that encloses the region bounded by $x=0$, $y=0$, and $y=x^2-1$, traversed counterclockwise.

   $\boxed{\dfrac{8}{15}}$

4. Suppose a particle travels one revolution clockwise around the circle $x^2+y^2=4$ under the force field $\mathbf{F}(x,y) = \langle y^3+\sin(x), e^y - x^3\rangle$. Find the work done by $\mathbf{F}$.

   $\boxed{24\pi}$

5. Find the divergence and curl of $\mathbf{F}(x,y,z) = xe^{yz}\,\mathbf{j} + x^3z\,\mathbf{k}$.

   $\boxed{\operatorname{div}\mathbf{F} = xze^{yz} + x^3, \qquad \operatorname{curl}\mathbf{F} = \langle -xye^z,\ -3x^2z,\ e^{yz}\rangle}$

6. Find the divergence and curl of $\mathbf{F}(x,y,z) = \langle xyz, 2\sin(xz), y^2z^3\rangle$.

   $\boxed{\operatorname{div}\mathbf{F} = yz + 3y^2z^2, \qquad \operatorname{curl}\mathbf{F} = \langle 2yz^3 - 2x\cos(xz),\ xy,\ 2z\cos(xz) - xz\rangle}$

7. Is $\mathbf{F}(x,y,z) = \langle 3x^2y+4y^2z, x^3+8xyz, 4xy^2-e^z\rangle$ a conservative vector field? If so, find a potential function for $\mathbf{F}$.

   $\boxed{\text{Yes, } f(x,y,z) = x^3y + 4xy^2z - e^z.}$

8. Find the area of the part of the plane given by $\mathbf{r}(u,v) = \langle 2+v, u-3v, 9-2u+v\rangle$, $-1 \le u \le 1$, $0 \le v \le 3$.

   $\boxed{6\sqrt{30}}$

   Video errata: At 1:45, I accidentally write magnitude when calculating a cross product. This is fixed around 3:07.

9. Find the area of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 49$.

$\boxed{\frac{2\pi}{3}\left(50^{3/2} - 1\right)}$

10. Find the area of the part of the plane $2x + 5y + z = 10$ that lies in the first octant.

$\boxed{5\sqrt{30}}$