Math 152 - Week-In-Review 9

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Convert the given parametric equations to Cartesian equations and find the corresponding graph.
   a. $x = -t^2$, $y = t + 1$, $-3 \leq t \leq 3$
   b. $x = \sqrt{t}$, $y = 1 - t$

2. Eliminate the parameter to obtain the Cartesian equation and the graph given by the following:
   a. $x = 6\cos(\theta)$, $y = 7\sin(\theta)$, $-\pi/2 \leq t \leq \pi/2$
   b. $x = \sin(t)$, $y = \csc(t)$, $0 < t < \pi/2$

3. Find the exact length of the curve given by $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \leq t \leq 2$.

4. Find the integral that represents the surface area obtained by rotating the following curve about the x-axis.
   $x = t\cos(t)$, $y = t\sin(t)$, $0 \leq t \leq \pi/2$

5. Find the exact area of the surface obtained by rotating the given curve about the y-axis.
   $x = 9t^2$, $y = 9t - 3t^3$, $0 \leq t \leq 2$

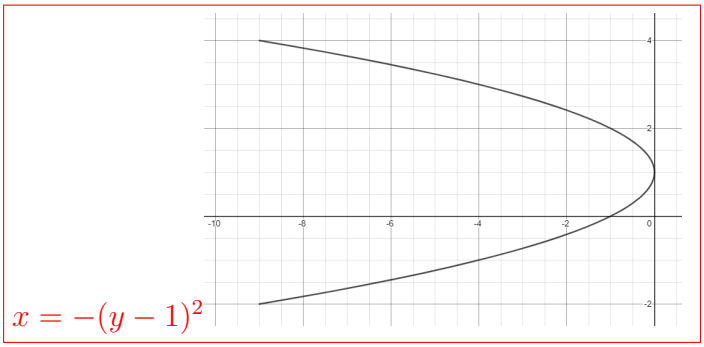
Solutions

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 9 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

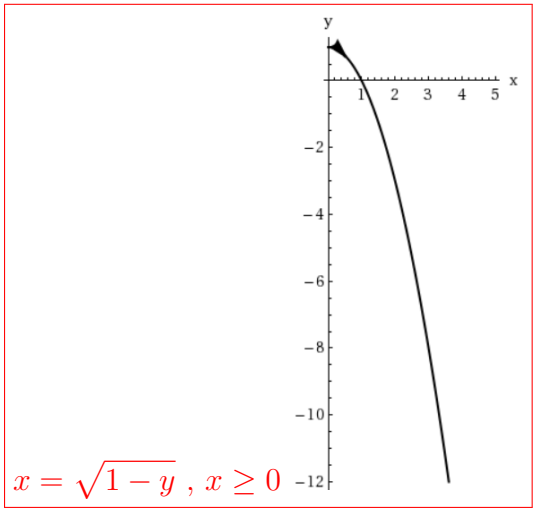
1. Convert the given parametric equations to Cartesian equations and find the corresponding graph.

   a. $x = -t^2$, $y = t + 1$, $-3 \le t \le 3$

   b. $x = \sqrt{t}$, $y = 1 - t$



$x = -(y-1)^2$

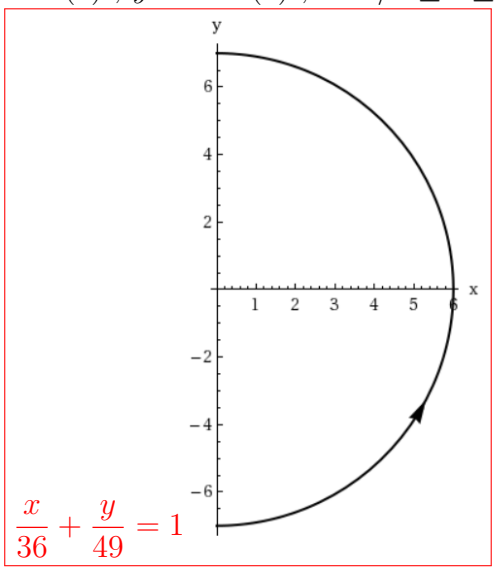


$x = \sqrt{1-y}$, $x \ge 0$

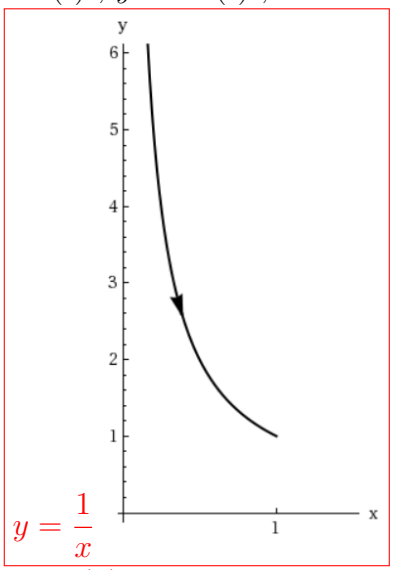
2. Eliminate the parameter to obtain the Cartesian equation and the graph given by the following:

   a. $x = 6\cos(\theta)$, $y = 7\sin(\theta)$, $-\pi/2 \le t \le \pi/2$

   b. $x = \sin(t)$, $y = \csc(t)$, $0 < t < \pi/2$



$\frac{x}{36} + \frac{y}{49} = 1$



$y = \frac{1}{x}$

Video errata: There should be an open circle at the point $(1, 1)$ in part (b).

3. Find the exact length of the curve given by $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \le t \le 2$.

$e^2 - e^{-2}$

4. Find the integral that represents the surface area obtained by rotating the following curve about the x-axis.
$x = t\cos(t)$, $y = t\sin(t)$, $0 \le t \le \pi/2$

$$2\pi \int_0^{\pi/2} t\sin(t)\sqrt{t^2+1}\,dt$$

Video errata: The Surface Area formula and the answer both need to be multiplied by 2π. Also the t inside the square root should be t^2.

5. Find the exact area of the surface obtained by rotating the given curve about the y-axis.
$x = 9t^2$, $y = 9t - 3t^3$, $0 \le t \le 2$

$$162\pi\left(\frac{8}{3} + \frac{32}{5}\right)$$