Math 152 - Week-In-Review 2

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. The base of a solid is the region enclosed by $x = y^2 - 9$ and $x = 7$. Its cross-sections are perpendicular to the x-axis and are equilateral triangles. Set up an integral to find the volume of the solid.

2. The base of a solid is the region enclosed by $y = \sqrt{x+5}$, $x = 4$ and the x-axis. Its cross-sections are perpendicular to the y-axis and are semicircles. Set up an integral to find the volume of the solid.

3. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the x-axis: $y = 2/x$, the x-axis, $x = 1$ and $x = 4$.

4. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the line $x = 7$: $x = y^2 + 3$ and $x = 7$.

5. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the y-axis: $y = \ln(x)$, the x-axis and $x = e$.

6. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the line $y = 5$: $y = \sqrt{x+1}$, the x-axis and $x = 8$.

7. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the line $x = 5$: $y = 3x - x^2$ and $y = 3x - 9$.

8. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the line $y = 5$: $y = \sqrt{x-4}$, the x-axis and $x = 8$.

9. Suppose a spring has natural length of 3 ft and it takes 10 ft-lb to stretch it from 5 ft to 8 ft.
   a. How much work is required to stretch the spring from 4 ft to 7 ft?
   b. How far beyond its natural length would a force of 3 lb keep the spring stretched?

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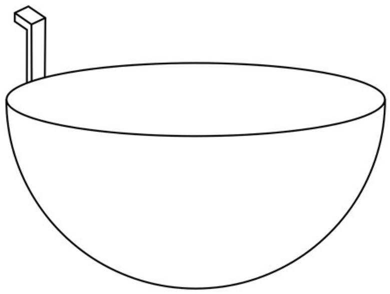
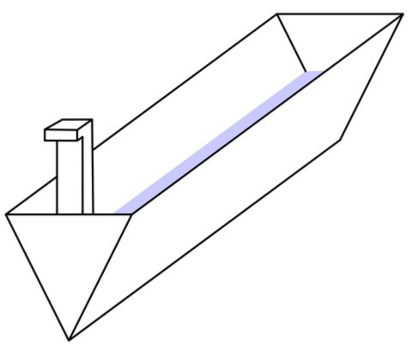
10. A hemispherical tank has a radius of 10 m with a 2 m spout at the top of the tank. The tank is filled with water to a depth of 7 m. Set up an integral that would compute the work required to pump all the water out of the spout. Use the fact that the weight density of water is 9800 N/m^3.



11. A trough with isosceles triangles as its ends is filled with water to a depth of 2 ft. The tank is 4 ft tall, 6 ft across at the top, 10 ft long and has a 3 ft spout. Set up an integral that would compute the work required to pump all the water out of the spout. Use the fact that the weight density of water is 62.5 lb/ft^3.



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SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 2 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. The base of a solid is the region enclosed by $x = y^2 - 9$ and $x = 7$. Its cross-sections are perpendicular to the x-axis and are equilateral triangles. Set up an integral to find the volume of the solid.

   An integral giving the volume is $\int_{-9}^{7} \sqrt{3}(x+9)\,dx$.

2. The base of a solid is the region enclosed by $y = \sqrt{x+5}$, $x = 4$ and the x-axis. Its cross-sections are perpendicular to the y-axis and are semicircles. Set up an integral to find the volume of the solid.

   An integral giving the volume is $\int_{0}^{3} \frac{1}{8}\pi(9-y^2)^2\,dy$.

3. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the x-axis: $y = 2/x$, the x-axis, $x = 1$ and $x = 4$.

   An integral giving the volume is $\int_{1}^{4} \frac{\pi x^2}{4}\,dx$.

4. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the line $x = 7$: $x = y^2 + 3$ and $x = 7$.

   An integral giving the volume is $\int_{-2}^{2} \pi\left(4-y^2\right)^2\,dy$.

5. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the y-axis: $y = \ln(x)$, the x-axis and $x = e$.

   An integral giving the volume is $\int_{0}^{1} \pi(e^2 - e^{2y})\,dy$.

   Video errata: I kept saying disk, but should have said washer.

6. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the line $y = 5$: $y = \sqrt{x+1}$, the x-axis and $x = 8$.

   An integral giving the volume is $\int_{-1}^{8} \pi\left[25 - (5-\sqrt{x+1})^2\right]\,dx$.

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7. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the line $x = 5$: $y = 3x - x^2$ and $y = 3x - 9$.

An integral giving the volume is $\int_{-3}^{3} 2\pi(9 - x^2)(5 - x)\,dx$.

Video errata: I said "Actually this is the volume of the shell" and wrote V, which is not true. It's the surface area!

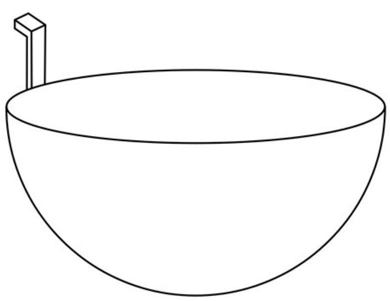
8. Set up an integral representing the volume of the solid obtained by rotating the region bounded by the following curves about the line $y = 5$: $y = \sqrt{x-4}$, the x-axis and $x = 8$.

An integral giving the volume is $\int_{0}^{2} 2\pi(5 - y)(4 - y^2)\,dy$.

9. Suppose a spring has natural length of 3 ft and it takes 10 ft-lb to stretch it from 5 ft to 8 ft.
a. How much work is required to stretch the spring from 4 ft to 7 ft?
b. How far beyond its natural length would a force of 3 lb keep the spring stretched?

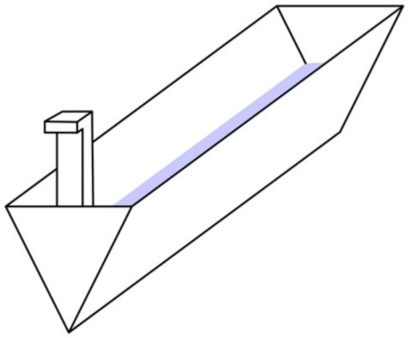
a. 150/21 ft·lb, b. 63/20 ft

10. A hemispherical tank has a radius of 10 m with a 2 m spout at the top of the tank. The tank is filled with water to a depth of 7 m. Set up an integral that would compute the work required to pump all the water out of the spout. Use the fact that the weight density of water is 9800 N/m^3.



The total work is given by $\int_{3}^{10} 9800\pi(100 - y^2)(y + 2)\,dy$.

11. A trough with isosceles triangles as its ends is filled with water to a depth of 2 ft. The tank is 4 ft tall, 6 ft across at the top, 10 ft long and has a 3 ft spout. Set up an integral that would compute the work required to pump all the water out of the spout. Use the fact that the weight density of water is 62.5 lb/ft^3.



The total work is given by $\int_2^4 62.5\,[15(4-y)]\,(y+2)\,dy$.

Video errata: When labeling the picture on the left (the one in 3D), I said to start the axes at the top of the water. This is not correct: start it at the top of the tank as I did in the right-hand drawing, looking at the tank from the side. Also, I used the wrong weight density: since we're working in ft, it needs to be 62.5 ft/lb^3, NOT 9800 N/m^3.

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