Math 152 - Week-In-Review 4

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Evaluate each of the following using trigonometric substitution.

   a. $\displaystyle\int \frac{x^2}{(4-9x^2)^{3/2}}\,dx$ b. $\displaystyle\int \frac{\sqrt{4x^2-25}}{x^4}\,dx$ c. $\displaystyle\int \frac{1}{(x+3)^2\sqrt{x^2+6x+5}}\,dx$

2. Evaluate each of the following using long division or partial fraction decomposition.

   a. $\displaystyle\int \frac{3x^2+4x+3}{x^2+1}\,dx$ c. $\displaystyle\int \frac{2x^3-4x^2+18x-11}{(x^2+9)(x^2+4)}\,dx$

   b. $\displaystyle\int \frac{x^2-4x+2}{(x+2)^2(x+1)}\,dx$

3. Determine if the following integrals converge or diverge. If it converges, compute the value.

   a. $\displaystyle\int_{-1}^{2} \frac{9}{(x+1)^2}\,dx$ c. $\displaystyle\int_{1}^{5} (x-1)\ln(x-1)\,dx$

   b. $\displaystyle\int_{0}^{\infty} \frac{5x^2}{\sqrt[3]{x^3+1}}\,dx$ d. $\displaystyle\int_{3}^{\infty} \frac{4}{3x^2+4x}\,dx$

4. Use the comparison test to determine if the following integrals converge or diverge.

   a. $\displaystyle\int_{1}^{\infty} \frac{3\sin^2(x)+2}{\sqrt{x}}\,dx$ c. $\displaystyle\int_{3}^{\infty} \frac{10}{\sqrt{x}+5x\sqrt{x}}\,dx$

   b. $\displaystyle\int_{2}^{\infty} \frac{\cos(x)+5}{\sqrt{x^3+7}}\,dx$ d. $\displaystyle\int_{1}^{\infty} \frac{2+e^{-x}}{x}\,dx$

SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 4 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Evaluate each of the following using integration-by-parts.

   General Guidance. Aim to replace a plus or minus squared algebraic term plus or minus a constant squared with a trig function using $\sin^2(\theta) + \cos^2(\theta) = 1$, or a variant of it such as $\tan^2(\theta) + 1 = \sec^2(\theta)$. This allows us to replace the sum or difference with a single trig function squared to simplify the integral.

   a. $\displaystyle\int \frac{x^2}{(4-9x^2)^{3/2}}\,dx = \boxed{\frac{1}{27}\left[\frac{3x}{\sqrt{4-9x^2}} - \arcsin\left(\frac{3x}{2}\right)\right] + C}$

   b. $\displaystyle\int \frac{\sqrt{4x^2-25}}{x^4}\,dx = \boxed{\frac{8}{75}\left[\frac{\sqrt{4x^2-25}}{5}\right]^3 + C}$

   c. $\displaystyle\int \frac{1}{(x+3)^2\sqrt{x^2+6x+5}}\,dx = \boxed{\frac{1}{4}\cdot\frac{\sqrt{(x+3)^2-4}}{x+3} + C}$

2. Evaluate each of the following using long division or partial fraction decomposition.

   a. $\displaystyle\int \frac{3x^2+4x+3}{x^2+1}\,dx = \boxed{3x + 2\ln|x^2+1| + C}$

   b. $\displaystyle\int \frac{x^2-4x+2}{(x+2)^2(x+1)}\,dx = \boxed{-6\ln|x+2| + \frac{14}{x+2} + 7\ln|x+1| + C}$

   c. $\displaystyle\int \frac{2x^3-4x^2+18x-11}{(x^2+9)(x^2+4)}\,dx = \boxed{-\frac{5}{3}\arctan(3x) + \ln|x^2+4| + \frac{1}{2}\arctan(2x) + C}$

3. Determine if the following integrals converge or diverge. If it converges, compute the value.

   a. $\displaystyle\int_{-1}^{2} \frac{9}{(x+1)^2}\,dx$ $\boxed{\text{diverges}}$

   b. $\displaystyle\int_{0}^{\infty} \frac{5x^2}{\sqrt[3]{x^3+1}}\,dx$ $\boxed{\text{diverges}}$

   c. $\displaystyle\int_{1}^{5} (x-1)\ln(x-1)\,dx$ $\boxed{= 8\ln(4) - 4}$

   d. $\displaystyle\int_{3}^{\infty} \frac{4}{3x^2+4x}\,dx$ $\boxed{= \ln\left(\frac{13}{9}\right)}$

4. Use the comparison test to determine if the following integrals converge or diverge.

a. $\displaystyle\int_1^{\infty} \frac{3\sin^2(x)+2}{\sqrt{x}}\,dx$ diverges

b. $\displaystyle\int_2^{\infty} \frac{\cos(x)+5}{\sqrt{x^3+7}}\,dx$ converges

c. $\displaystyle\int_3^{\infty} \frac{10}{\sqrt{x}+5x\sqrt{x}}\,dx$ converges

d. $\displaystyle\int_1^{\infty} \frac{2+e^{-x}}{x}\,dx$ diverges