



## MATH 152 - WEEK-IN-REVIEW 5

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### PROBLEM STATEMENTS

You should attempt the problems yourself first. The next section contains the solutions.

1. Find a formula for the general term,  $a_n$ , of the sequence assuming that the pattern of the first few terms continues. Give the formula so that the first term is  $a_1$ .

$$\left\{ \frac{5}{27}, -\frac{9}{64}, \frac{13}{125}, -\frac{17}{216}, \dots \right\}$$

2. Determine whether each sequence converges or diverges. If it converges, find the limit.

a.  $a_n = \frac{\ln(\ln(2n))}{5n}$       b.  $a_n = \frac{(-1)^n n^3}{7n^3 - 1}$       c.  $a_n = \arctan\left(\frac{1 - n^2}{3n + 1}\right)$

3. Determine whether the following sequence are increasing, decreasing, or not monotonic. Also determine if the sequence is bounded.

a.  $a_n = \ln(8 + 9n) - \ln(3n - 2)$       b.  $a_n = \tan\left(\frac{(2n - 1)\pi}{4}\right)$

4. The sequence below is bounded and increasing (check!). Does it converge and if so, what to?

$$a_1 = 3 \qquad a_{n+1} = \frac{5a_n - 4}{a_n}$$

5. Determine if the series converges or diverges. Give the sum if it converges.

a.  $\sum_{k=1}^{\infty} k^2 \sin\left(\frac{1}{k^2}\right)$       b.  $\sum_{n=1}^{\infty} \frac{(-1)^n - 3^n}{7^n}$       c.  $\sum_{k=1}^{\infty} (e^{4/(k+1)} - e^{4/k})$

6. Consider:  $\sum_{n=1}^{\infty} 3n^2 e^{-n^3}$ .

- a. Show that the series converges.  
b. Estimate the maximum error involved by approximating the series using the first eight terms.  
c. At least how many terms are required to estimate the series within  $e^{-27}$  error?

## SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the [Week 5 playlist \(clickable link\)](#). You can turn on closed captions by clicking “CC” inside YouTube as well as adjust the video speed inside of “Settings” by clicking the cog in the bottom right of the player.

1. Find a formula for the general term,  $a_n$ , of the sequence assuming that the pattern of the first few terms continues. Give the formula so that the first term is  $a_1$ .

$$\left\{ \frac{5}{27}, -\frac{9}{64}, \frac{13}{125}, -\frac{17}{216}, \dots \right\}$$

$$a_n = \frac{(-1)^{n+1}(4n+1)}{(n+2)^3}$$

2. Determine whether each sequence converges or diverges. If it converges, find the limit.

a.  $a_n = \frac{\ln(\ln(2n))}{5n}$

converges to 0

b.  $a_n = \frac{(-1)^n n^3}{7n^3 - 1}$

diverges

c.  $a_n = \arctan\left(\frac{1-n^2}{3n+1}\right)$

converges to  $-\pi/2$

3. Determine whether the following sequence are increasing, decreasing, or not monotonic. Also determine if the sequence is bounded.

(a)  $a_n = \ln(8+9n) - \ln(3n-2)$

bounded and decreasing

(b)  $a_n = \tan\left(\frac{(2n-1)\pi}{4}\right)$

bounded and not monotonic

**Video errata:** I spoke about convergence in the video rather than monotonicity. Not only does it not converge, but it is not monotonic. This is because no matter how far out we go in the sequence, it will still continue alternating between two fixed numbers,  $-1$  and  $1$ .

4. The sequence below is bounded and increasing (check!). Does it converge and if so, what to?

$$a_1 = 3 \qquad a_{n+1} = \frac{5a_n - 4}{a_n}$$

It converges to 4.



5. Determine if the series converges or diverges. Give the sum if it converges.

a.  $\sum_{k=1}^{\infty} k^2 \sin\left(\frac{1}{k^2}\right)$  diverges

b.  $\sum_{n=1}^{\infty} \frac{(-1)^n - 3^n}{7^n}$  converges to  $-\frac{7}{8}$

c.  $\sum_{k=1}^{\infty} (e^{4/(k+1)} - e^{4/k})$  converges to  $1 - e^4$

**Video errata:** I said converge towards the end of the video due to the inside of the series converging to something not equal to zero, but should have said diverges.

6. Consider:  $\sum_{n=1}^{\infty} 3n^2 e^{-n^3}$ .

- a. Show that the series converges.
- b. Estimate the maximum error involved by approximating the series using the first eight terms.
- c. At least how many terms are required to estimate the series within  $e^{-27}$  error?

a. Converges to  $e^{-1}$       b. bounded above by  $e^{-512}$       c. only 3 terms