Math 152 - Week-In-Review 6

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Problem Statements

You should attempt the problems yourself first. The next section contains the solutions.

1. Use the comparison test to determine if the series converges or diverges.

a. $\displaystyle\sum_{n=5}^{\infty} \frac{7^n - 8}{9 + 5^n}$ b. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}}$ c. $\displaystyle\sum_{n=1}^{\infty} \frac{n\ln(n)}{2n^5 + \cos(n)}$ d. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^3e^{-5n} + 1}{7n^6 + 3n}$

2. Use the limit comparison test to determine if the series converges or diverges.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{9n^3 + 2}{7n^4 - \sin(5n)}$ b. $\displaystyle\sum_{n=3}^{\infty} \frac{\sqrt{5n - 6}}{3n^3 + 1}$ c. $\displaystyle\sum_{n=2}^{\infty} \frac{7n^6 + n^4}{\sqrt[3]{5n^2 - 1 + n^{15}}}$

3. Use the alternating series test to determine if the series converges or diverges.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{5\sin((n + 3/2)\pi)}{9 + \sqrt{n}}$ b. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n + 2)}{3n^2 + 4n}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{2n+1}\cos(n\pi)}{\sqrt[3]{n}}$ d. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{\sqrt{n^4 + 2}}$

4. Consider the 4^{th} partial sum of the series: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n n!}$. Use the alternating series test to obtain an upper bound on the absolute value of the error.

5. How many terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(n + 3)^{1/5}}$ is needed to find a partial sum that approximates the series to withing 0.0001 accuracy?

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Solutions

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the Week 6 playlist (clickable link). You can turn on closed captions by clicking "CC" inside YouTube as well a adjust the video speed inside of "Settings" by clicking the cog in the bottom right of the player.

1. Use the comparison test to determine if the series converges or diverges.

   a. $\displaystyle\sum_{n=5}^{\infty} \frac{7^n - 8}{9 + 5^n}$ — diverges

   b. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}}$ — diverges

   c. $\displaystyle\sum_{n=1}^{\infty} \frac{n\ln(n)}{2n^5 + \cos(n)}$ — converges

   d. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^3 e^{-5n} + 1}{7n^6 + 3n}$ — converges

   Video errata: In Part a., I should have shown that $9 + 5^n$ is less than something, not greater than, to make the sum smaller since we're dividing by it. Rather than $9 + 5^n \geq 5^n$, I should have shown $9 + 5^n \leq 2 \cdot 5^n$. This follows from $9 < 5^5 \leq 5^n$ for $n \geq 5$ and so $9 + 5^n < 5^n + 5^n = 2 \cdot 5^n$. Note the final answer is the same in that the series diverges, but with $1/4$ in front instead of $1/2$.

   In Part c., $\cos(1)$ should be $\cos(n)$. The argument still works.

2. Use the limit comparison test to determine if the series converges or diverges.

   a. $\displaystyle\sum_{n=2}^{\infty} \frac{9n^3 + 2}{7n^4 - \sin(5n)}$ — diverges

   b. $\displaystyle\sum_{n=3}^{\infty} \frac{\sqrt{5n - 6}}{3n^3 + 1}$ — converges

   c. $\displaystyle\sum_{n=2}^{\infty} \frac{7n^6 + n^4}{\sqrt[3]{5n^2 - 1 + n^{15}}}$ — diverges

3. Use the alternating series test to determine if the series converges or diverges.

   a. $\displaystyle\sum_{n=1}^{\infty} \frac{5\sin((n + 3/2)\pi)}{9 + \sqrt{n}}$ — converges

   b. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n + 2)}{3n^2 + 4n}$ — converges

   c. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{2n+1} \cos(n\pi)}{\sqrt[3]{n}}$ — converges

   d. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{\sqrt{n^4 + 2}}$ — diverges

   Video errata: In Part a., I didn't check that $b_n \geq 0$, which you must do to apply the AST. In this case it's obvious since $\sqrt{n} > 0$, but you should really state it.

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4. Consider the 4^{th} partial sum of the series: $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n n!}$. Use the alternating series test to obtain an upper bound on the absolute value of the error.

It's bounded by $\frac{1}{e^5 \cdot 5!} \approx 0.000056$. Answers can vary.

5. How many terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+3)^{1/5}}$ is needed to find a partial sum that approximates the series to withing 0.0001 accuracy?

It'd take at least $10000^5 - 4 = 99999999999999999996$ terms. This is very impractical!

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