



## MATH 152 - WEEK-IN-REVIEW 8

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### PROBLEM STATEMENTS

You should attempt the problems yourself first. The next section contains the solutions.

1. Find the power series representation of  $f(x)$  centered around  $x = 0$  and the radius of convergence.

a.  $f(x) = -\frac{7x^2}{(1+2x)^3}$                       b.  $f(x) = \ln(13 - 3x^4)$

2. Evaluate the indefinite integral,  $\int \frac{\arctan(5x^4)}{x^3} dx$ , using a power series.

3. Find the Maclaurin series of  $f(x) = x^2 \cos^2(3x)$ .

4. Express  $\int_0^{2/3} x^3 \sin(x^2) dx$  as an infinite series.

5. Find the Taylor series of  $f(x) = \frac{3}{(4x-5)^6}$  centered at  $x = 2$ .

6. Find the Taylor series of  $f(x) = (x^2 - x)e^x$  centered at  $x = 1$ .

7. Find the 4<sup>th</sup> degree Taylor polynomial,  $T_4(x)$ , for  $f(x) = e^{-3x} - 2 - 3x$  centered at  $x = 4$ .

## SOLUTIONS

Click the boxed answer (also in red) to watch the video solution. Note any video errata. You can also see them all by viewing the [Week 8 playlist \(clickable link\)](#). You can turn on closed captions by clicking “CC” inside YouTube as well as adjust the video speed inside of “Settings” by clicking the cog in the bottom right of the player.

1. Find the power series representation of  $f(x)$  centered around  $x = 0$  and the radius of convergence.

a.  $f(x) = -\frac{7x^2}{(1+2x)^3}$

b.  $f(x) = \ln(13 - 3x^4)$

$$\sum_{n=2}^{\infty} \frac{-7(-2)^n n(n-1)x^n}{8}, R = \frac{1}{2}$$

$$\ln(13) - \sum_{n=0}^{\infty} \left(\frac{3}{13}\right)^{n+1} \frac{x^{4n+4}}{n+1}, R = \sqrt[4]{\frac{13}{3}}$$

2. Evaluate the indefinite integral,  $\int \frac{\arctan(5x^4)}{x^3} dx$ , using a power series.

$$\left( \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1}}{(2n+1)(8n+2)} x^{8n+2} \right) + C$$

**Video errata:** I forgot the  $+C$  in the video, but it's necessary because the integral is indefinite!

3. Find the Maclaurin series of  $f(x) = x^2 \cos^2(3x)$ .

$$\frac{x^2}{2} + \sum_{n=0}^{\infty} \frac{(-1)^n 6^{2n}}{2 \cdot (2n)!} x^{2n+2}$$

4. Express  $\int_0^{2/3} x^3 \sin(x^2) dx$  as an infinite series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+6)(2n+1)!} \left(\frac{2}{3}\right)^{4n+6}$$

5. Find the Taylor series of  $f(x) = \frac{3}{(4x-5)^6}$  centered at  $x = 2$ .

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot (n+1)(n+2)(n+3)(n+4)(n+5) \cdot 4^n}{120 \cdot 3^{n+5}} (x-2)^n$$



6. Find the Taylor series of  $f(x) = (x^2 - x)e^x$  centered at  $x = 1$ .

$$\sum_{n=0}^{\infty} \frac{n^2 e}{n!} (x - 1)^n$$

7. Find the 4<sup>th</sup> degree Taylor polynomial,  $T_4(x)$ , for  $f(x) = e^{-3x} - 2 - 3x$  centered at  $x = 4$ .

$$T_4(x) = (e^{-12} - 14) - (3e^{-12} + 3)(x - 4) + \frac{9}{2}e^{-12}(x - 4)^2 - \frac{9}{2}e^{-12}(x - 4)^3 + \frac{27}{8}e^{-12}(x - 4)^4$$