

Math 148 Help Session Interview Questions

1. Evaluate

$$\int \frac{4x^2 + 13x - 9}{x^3 + 2x^2 - 3x} dx.$$

2. Determine whether the infinite integral below is convergent or divergent.

$$\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$$

3. Solve the differential equation $y' = y^2 \sin x = 0$ with initial condition $y(0) = \frac{1}{3}$.
4. Assume that the size of a population evolves according to the logistic equation with intrinsic growth rate $r = 1.25$ and carrying capacity $K = 500$.
- Find the differential equation that describes the growth rate of this population.
 - Find all equilibria and discuss the stability of the equilibria using the graphical approach.
 - Find the eigenvalues associated with the equilibria and use the stability criterion to determine stability of the equilibria.

5. Solve the linear system

$$\begin{aligned}x + 2y - 3z &= -2 \\3x - y - 2z &= 1 \\2x + 3y - 5z &= -3\end{aligned}$$

by performing the augmented matrix and performing Gaussian elimination.

6. Let $A = (0, 1, -3)$ and $B = (1, 3, -2)$. Find the unit vector in the direction of \overrightarrow{AB} .

7. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} -4 & 2 \\ -3 & 1 \end{bmatrix}.$$

8. Find the equation of the tangent plane to $f(x, y) = \sqrt{4 - x^2 - 2y^2}$ at $(1, -1, 1)$.
9. Find all local extrema and saddle points of $f(x, y) = -x^2 - y^2 + 6x + 8y - 21$.
10. Find all nonnegative equilibria of

$$\begin{aligned}x_1(t+1) &= 2x_1(t)[1 - x_1(t)] \\x_2(t+1) &= x_1(t)[1 - x_2(t)]\end{aligned}$$

and determine their stability.

11. Consider the linear system

$$\begin{aligned}\frac{dx_1}{dt} &= 2x_1 - 3x_2, \\ \frac{dx_2}{dt} &= 2x_1 - x_2.\end{aligned}$$

Determine the stability of $(0, 0)$ and classify the equilibrium.

12. Find all equilibria of the nonlinear system

$$\begin{aligned}\frac{dx_1}{dt} &= 4x_1(1 - x_1) - 2x_1x_2, \\ \frac{dx_2}{dt} &= x_2(2 - x_2) - x_2.\end{aligned}$$

Determine the stability of each equilibrium.