Math 251 Help Session Interview Questions

1. Consider the surface $xyz + 2x + 3y + 3z + 2 = 0$

   (a) Find an equation of the tangent plane to the surface at the point $(1, 2, -2)$.

   (b) Find parametric equations of the normal line to the surface at this same point.

2. Find all maxima, minima, and saddle points, if any, for $f(x, y) = x^3 + 3xy + y^3$.

3. Find the absolute maxima and minima of the function $f(x, y) = x^2 + xy + y^2 - 6x + 2$ on the region $0 \leq x \leq 1$, $-3 \leq y \leq 0$.

4. Let $f(x, y, z) = \ln(2x + 3y + 6z)$. Find the unit vector in the direction in which f decreases most rapidly at the point $(-1, -1, 1)$ and find the rate of change of f in this direction.

5. Given that $w = \sin(2x - y), x = r + \tan x$, and $y = rs^2$, find $\frac{\partial w}{\partial r}$ in terms of r and s.

6. Find the volume of the solid lying under the paraboloid $z = x^2 + 4y^2$ and above the square $R = [1, 2] \times [0, 1]$.

7. Evaluate $\displaystyle\int_0^4 \int_{y/2}^2 e^{x^2}\, dx\, dy$.

8. A particle moves along the curve parametrized by $\mathbf{r}(t) = t\mathbf{i} - t^2\mathbf{j} + t^3\mathbf{k}$, $0 \leq t \leq 1$, under a force given by $\mathbf{F}(x, y, z) = 2z\mathbf{i} - y\mathbf{j} + x\mathbf{k}$. Calculate the work done on the particle by the force.

9. (a) Show that the vector field $\mathbf{F}(x, y) = (xy^2 + 3x^2y)\mathbf{i} + (x + y)x^2\mathbf{j}$ is conservative.

   (b) Find the potential function.

   (c) Evaluate the integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where C consists of line segments from $(1, 1)$ to $(0, 2)$ to $(3, 0)$.

10. Set up a triple integral to find the volume of the solid bounded by the surfaces $x = 0$, $y = 0$, $z = 0$, and $x + 2y + 3z = 1$.

11. Sketch the solid whose volume in cylindrical coordinates is given by $\displaystyle\int_0^{2\pi} \int_0^2 \int_{r^2}^4 r\, dz\, dr\, d\theta$ and find the volume.

12. Set up a triple integral in spherical coordinates to find the volume of the region below the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z^2 = x^2 + y^2$.

13. Evaluate the line integral $\displaystyle\int_C yz\, ds$ where C is the curve with parametrization $\mathbf{r}(t) = t\mathbf{i} + 3t\mathbf{j} + 2t\mathbf{k}$, $1 \leq t \leq 2$.

14. Use Green's Theorem to evaluate $\displaystyle\int_C -y^3\, dx + x^3\, dy$, where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

15. Use Stokes' Theorem to evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle z^2, y^2, xy \rangle$ and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 2)$, oriented counterclockwise.

16. Evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle 3x, 3y, 3z \rangle$ and S is the surface of the unit sphere.