MATH 308 Sample Problems

Kamran Reihani

September 2, 2020

1. Consider the differential equation $y(3+xy)\,dx - 3x\,dy = 0$.
   (a) Show that the equation is not exact and find an integrating factor which will make the equation exact.
   (b) Find the general solution of the equation.
   (c) Solve the initial value problem (IVP) with $y(1) = 2$.
   (d) Express your solution of the IVP in explicit form.
   (e) Find the interval of validity of the solution of the IVP.
2. Find the general solution of the differential equation
$$x^2y'' + xy' + (x^2 - \frac{1}{4})y = x^{\frac{3}{2}},$$
given that $y_1 = x^{-\frac{1}{2}}\cos x$ is a solution of the corresponding homogeneous equation.
3. Find the general solution of the differential equation $2\ddot{y} + 4\dot{y} + 7y = 343\,t^2$.
4. Solve the initial value problem
$$\ddot{y} - 5\dot{y} + 4y = 2\delta(t-1) + 5u_3(t),\ y(0) = 0,\ y'(0) = 3.$$
5. Find the general solution of the system of differential equations
$$\dot{\mathbf{x}} = \begin{pmatrix} -4 & -2 \\ 6 & 3 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ 3e^{2t} \end{pmatrix}.$$