Math 151 - Week-In-Review 13

Topics for the week:

5.3 The Fundamental Theorem of Calculus
Some Exam 1 Reminders

5.3 The Fundamental Theorem of Calculus

1. Use the Fundamental Theorem of Calculus to determine the derivative of $f(x) = \int_0^{\sqrt{x}} (\cos(t))\ dt$.

2. Compute $\frac{d}{dx}\left[\int_1^{x^4} \left(\sqrt{t^3+3t-1}\right)\ dt\right]$.

3. Determine the derivative of $y = \int_{\tan(\theta)}^{4} \left(\sec^5(x) - 6x^3\right)\ dx$.

4. Compute the derivative of $\dfrac{d}{dx}\left[\int_{6x^2}^{5x^4} \left(\dfrac{2t}{1+t^2}\right)\ dt\right]$.

5. Compute $\int_0^4 \left(3x - \frac{x^3}{4}\right)\, dx$.

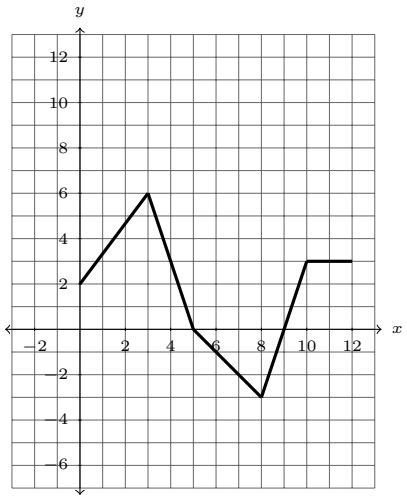
6. Compute $\int_{\frac{\pi}{2}}^0 \left(\cos^2(\theta)\right)\, d\theta$.

7. Compute $\displaystyle\int_1^{\sqrt{2}} \left(\frac{x^2+\sqrt{x}}{x^2}\right)\, dx.$

8. Compute $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(8y^2 - \sin(y)\right)\, dy.$

9. Given the initial value problem $\dfrac{dy}{dx} = \dfrac{1}{x}$, $y(\pi) = -3$; write a function that will solve this problem.

10. Let $g(x) = \displaystyle\int_0^x (f(t))\ dt$, where $f(x)$ is the function whose graph is shown below.



(a) Evaluate $g(5)$

(b) Evaluate $g(12)$

(c) On what interval is g decreasing?

(d) Does $g(x)$ have a maximum, and if so, at what value of x?

E1 Some Reminders from Exam 1

11. Given $\mathbf{a} = \langle 2, 5 \rangle$ and $\mathbf{b} = 8\mathbf{i} - 6\mathbf{j}$,

    (a) Compute $-\mathbf{a} + 3\mathbf{b}$.

    (b) Compute a vector, $\vec{v}$, in the direction of $\mathbf{a}$, but of length 4.

    (c) Compute $\mathbf{a} \cdot \mathbf{b}$.

    (d) Determine the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$.

12. Given a force with magnitude 5 N is applied to an object at a 60° angle, moves the object 3 m; compute the work done.

13. Given $f(x) = \begin{cases} x^2 - 5a & \text{if} \quad x < -1 \\ ax^2 & \text{if} \quad -1 \leq x \leq 2 \\ 3ax + b & \text{if} \quad x > 2 \end{cases}$, determine the values for a and b that make $f(x)$ a continuous function over $(-\infty, \infty)$.

14. Compute each limit:

(a) $\lim_{x \to 2^+} \left(\dfrac{x+1}{x^2+2x-8} \right)$

(b) $\lim_{x \to 1^-} \left(\dfrac{x^2+3x-4}{|x-1|} \right)$

(c) $\lim_{x \to -\infty} \left(\dfrac{5-4x}{\sqrt{9x^2+2x}} \right)$

(d) $\lim_{x \to \infty} \left(\dfrac{-5e^{-x}+2e^{x}}{3e^{x}+8e^{-x}} \right)$