Math 151 - Week-In-Review 3

Topics for the week:

2.3 Calculating Limits Using Limit Laws
2.5 Continuity
2.6 Limits at Infinity and Horizontal Asymptotes

2.2 Cont. Limits

1. Evaluate the limit, $\lim_{x\to 3^-}\left(\frac{1-x}{2x-6}\right)$, if the limit exists.

2.3 Calculating Limits Using Limit Laws

2. Given $\lim_{x\to 4}(f(x)) = -8$, $\lim_{x\to 4}(g(x)) = 22$, $\lim_{x\to 4}(2h(x)) = 6$, and $\lim_{x\to 4^-}(k(x)) = 0$, determine the value of each limit, if the limit exists.

   (a) $\lim_{x\to 4}\left(\frac{\sqrt{g(x)+h(x)}}{3f(x)}\right) =$

   (b) $\lim_{x\to 4^-}\left(\frac{1}{2}f(x) - k(x)\right) =$

3. Evaluate the limit, $\lim_{x\to 0}\left((x+1)^{1996}\right)$, if it exists.

4. Evaluate the limit, $\lim_{t\to 3}\left(\dfrac{t^2-2t-3}{t^2-9}\right)$, if it exists.

5. Evaluate the limit, $\lim_{x\to -1}\left(\dfrac{x^2-1}{x^2-5x+6}\right)$, if it exists.

6. Evaluate the limit, $\lim_{h\to 0}\left(\frac{\sqrt{3h+7}-\sqrt{7}}{h}\right)$, if it exists.

7. Evaluate the limit, $\lim_{x\to -6}\left(\frac{x^{-1}+\frac{1}{6}}{x+6}\right)$, if it exists.

8. For $f(x) = \dfrac{|x-9|}{2x^2 - 17x - 9}$, evaluate $\lim\limits_{x \to 9^-} f(x)$ and $\lim\limits_{x \to 9^+} f(x)$. Does $\lim\limits_{x \to 9} f(x)$ exist?

9. For $f(x) = \begin{cases} x^2 - 36 & x < 6 \\ \ln(x-5) & x \geq 6 \end{cases}$, evaluate $\lim\limits_{x \to 6^-} f(x)$ and $\lim\limits_{x \to 6^+} f(x)$. Does $\lim\limits_{x \to 6} f(x)$ exist?

10. Evaluate the limit, $\lim_{x \to -\pi^+} \left((x+\pi)\sin\left(\frac{1}{x+\pi}\right)\right)$, if it exists.

2.5 Continuity

11. Determine whether $f(x) = \begin{cases} \frac{2x^2+x-6}{x+2} & \text{if } x < -2 \\ -7 & \text{if } x = -2 \\ \sqrt{3x+6} & \text{if } x > -2 \end{cases}$ is continuous at $x = -2$.

12. Determine the values where $f(x) = \dfrac{x^2 + 2x}{x^4 - 3x^3 - 10x^2}$ is not continuous. Then classify the value(s) as a vertical asymptote or removable discontinuity.

13. Determine the value(s) of a and b that will make $g(x) = \begin{cases} \frac{x^2+3x-10}{x-2} & \text{if } x < 2 \\ 4ax - 3b & \text{if } 2 \le x < 5 \\ e^{5-x} & \text{if } x \ge 5 \end{cases}$

a continuous function.

Use the Intermediate Value Theorem to show that there is a real number a in $(0, 2)$ such that $f(a) = 12$ for $f(x) = -x^4 + 3x^3 + 5$.

2.6 Limits at Infinity and Horizontal Asymptotes

14. Compute each of the following limits.

(a) $\lim_{x \to -\infty} (\arctan(x)) =$

(b) $\lim_{x \to \infty} \left(6x^4 - 7x^9\right) =$

15. Evaluate the limit, $\lim_{t\to\infty}\left(\frac{t^3-2t-3}{t^2-9}\right)$, if it exists. Then identify any horizontal asymptotes for the function.

16. Evaluate the limit as x approaches $-\infty$ for $h(x) = \frac{2e^{3x}+e^{-2x}}{3e^{4x}+5e^{-2x}}$. Then identify any horizontal asymptotes for the function.

17. Identify any horizontal asymptotes for the function $f(x) = \dfrac{\sqrt{2x^2+3}+5x}{x+2}$ by evaluating the given limits.

$$\lim_{x\to\infty}\left(\frac{\sqrt{2x^2+3}+5x}{x+2}\right)$$

$$\lim_{x\to-\infty}\left(\frac{\sqrt{2x^2+3}+5x}{x+2}\right)$$