Week in Review
Math 152

Week 15

Final Test Review

Substitution rule

$$\int_{-3}^{1} x\sqrt{x+3}\,dx =$$

(a) $\frac{-16}{5}$

(b) $\frac{32}{5}$

(c) $\frac{112}{15}$

(d) $\frac{-56}{5}$

(e) $\frac{-116}{5}$

Area between curves

Which of the following integrals gives the area of the region bounded by the curves $x = y^2$ and $x = 6 - y$?

(a) $\int_{-3}^{2} (6 - y - y^2)\, dy$

(b) $\int_{-3}^{2} (y^2 - 6 + y)\, dy$

(c) $\int_{4}^{9} (6 - x - \sqrt{x})\, dy$

(d) $\int_{4}^{9} (\sqrt{x} - 6 + x)\, dy$

(e) $\int_{4}^{9} (6 - y - y^2)\, dy$

General slicing method for any volume

Consider the solid whose base is the upper half of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$. Cross sections perpendicular to the y axis are semicircles. Find the volume of the solid.

Volumes of Solids of Revolution (washer)

Consider the region R bounded by $y = 2x^2$ and $y = 1$, first quadrant only.
Find the volume obtained by rotating R about the y-axis.

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) π

(d) $\frac{4\pi}{5}$

(e) None of the above

Volumes of Solids of Revolution (washer)

If we revolve the region bounded by $y = 1 - x^2$ and $x - y = 1$ about the line $y = 3$, which of the following integrals gives the resulting volume?

(a) $\int_{-1}^{2} 2\pi(3 - x)(x^2 - x + 2)\, dx$

(b) $\int_{-2}^{1} \pi\left((2 + x^2)^2 - (4 - x)^2\right)\, dx$

(c) $\int_{-1}^{2} 2\pi(x - 3)(x^2 - x + 2)\, dx$

(d) $\int_{-2}^{1} \pi\left((4 - x)^2 - (2 + x^2)^2\right)\, dx$

(e) $\int_{-1}^{2} \pi\left((2 + x^2)^2 - (4 - x)^2\right)\, dx$

Volumes of Solids of Revolution (shell)

Consider the region bounded by the two curves $y = \cos x$, $y = \sin x$ and the two lines $x = 0$ and $x = \dfrac{\pi}{4}$.
Which of the following represents the volume of this region being rotated about the line $x = -1$?

(a) $\displaystyle\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\cos x - \sin x)\,dx$ $\leftarrow$ correct

(b) $\displaystyle\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\sin x - \cos x)\,dx$

(c) $\displaystyle\int_{-1}^{\frac{\pi}{4}} 2\pi(x+1)(\cos x - \sin x)\,dx$

(d) $\displaystyle\int_0^{\frac{\pi}{4}} 2\pi(x+1)(\cos^2 x - \sin^2 x)\,dx$

(e) $\displaystyle\int_0^{\frac{\pi}{4}} \pi(\cos^2 x - \sin^2 x)\,dx$

Volumes of Solids of Revolution (shell)

Consider the region bounded by the curves $x = y^2 - 2y$ and the y-axis. Which of the following represents the volume of solid formed when the region is rotated about $y = 4$?

(a) $\int_0^2 2\pi y(y^2 - 2y)\, dy$

(b) $\int_0^2 2\pi y(2y - y^2)\, dy$

(c) $\int_0^2 2\pi(4 - y)(y^2 - 2y)\, dy$

(d) $\int_0^2 \pi(y - 4)(4y^2 - y^4)\, dy$

(e) $\int_0^2 2\pi(4 - y)(2y - y^2)\, dy$ $\leftarrow$ correct

Work (Spring)

11. An ideal spring has a natural length of 10 meters. The work done in stretching the spring from 14 meters to 18 meters is 24J. Determine the spring constant k.

    (a) $k = \frac{1}{2}\,\mathrm{N/m}$
    (b) $k = \frac{3}{8}\,\mathrm{N/m}$
    (c) $k = 1\,\mathrm{N/m}$ $\leftarrow$ correct
    (d) $k = 3\,\mathrm{N/m}$
    (e) $k = 6\,\mathrm{N/m}$

Work (Cable)

12. A 90 ft cable weighing 10 lb is hanging down the side of a 200 ft building. How much work is required to pull the rope 30 feet up the side of the building?

    (a) 6000 ft-lb
    (b) 1500 ft-lb
    (c) 250 ft-lb ← correct
    (d) 300 ft-lb
    (e) 50 ft-lb

Work (Pump)

A hemispherical tank has a radius of 10 meters with a 10 meters spout at the top of the tank. The tank is filled with water to a depth of 7 meters. The weight density of water is $\rho g\ N/m^3$. Suppose we want to find the work required to pump the water through the spout

Integration by parts

15. Evaluate $\int_0^1 \frac{x^2}{e^x}\,dx$.

(a) $2 - \frac{5}{e}$ ← correct

(b) $\frac{5}{e} - 2$

(c) $1 - \frac{3}{e}$

(d) $1 - \frac{2}{e}$

(e) $1 - \frac{1}{e}$

Trigonometric Integrals

Compute $\int_0^{\pi/2} \sin^2(\theta)\cos^3(\theta)\,d\theta$.

(a) $\frac{2}{5}$

(b) $\frac{4}{5}$

(c) $\frac{2}{15}$ $\leftarrow$ correct

(d) $\frac{8}{5}$

(e) None of the above

Trigonometric Integrals

Which of the following is equal to $\int_0^{\pi/4} \tan^2(\theta)\sec^4(\theta)\,d\theta$?

(a) $\int_0^{\pi/4} u^2(u^2-1)\,du$

(b) $\int_0^{\pi/4} u^2(1+u^2)\,du$

(c) $\int_0^{\sqrt{2}/2} u^2(1+u^2)\,du$

(d) $\int_0^{1} u^2(u^2-1)\,du$

(e) $\int_0^{1} u^2(1+u^2)\,du$

Trigonometric Substitution

After an appropriate substitution, the integral $\int \sqrt{x^2 + x}\, dx$ is equivalent to which of the following?

(a) $\int \tan^2\theta \sec\theta \, d\theta$

(b) $\frac{1}{4}\int \sec^3\theta \, d\theta$

(c) $-\frac{1}{4}\int \sin^2\theta \, d\theta$

(d) $\frac{1}{4}\int \tan^2\theta \sec\theta \, d\theta$

(e) $\int \cos^2\theta \, d\theta$

Integration by Partial Fractions

$$\int \frac{x^3+x}{x-1}\,dx =$$

(a) $\frac{x^3}{3}+\frac{x^2}{2}+2x-2\ln|x-1|+C$

(b) $\frac{x^3}{3}-\frac{x^2}{2}+2x+2\ln|x-1|+C$

(c) $\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln|x-1|+C$

(d) $\frac{x^3}{3}+\frac{x^2}{2}-2x+2\ln|x-1|+C$

(e) $\frac{x^3}{3}-\frac{x^2}{2}+2x-2\ln|x-1|+C$

Integration by Partial Fractions

8. Given the partial fraction decomposition $\dfrac{2x^2 - x + 4}{x(x^2+4)} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2+4}$, what are the values of A, B and C?

(a) A= 1, B= 1, C= −1.

(b) A= 4, B= −2, C= −1.

(c) A= 1, B= 1, C= 1.

(d) A= 4, B= 2, C= 1.

(e) A= 1, B= 2, C= 4.

Improper integrals

$$\int_1^\infty xe^{-x^2}\,dx =$$

(a) 1

(b) $2e$

(c) $\dfrac{1}{2e}$ $\leftarrow$ correct

(d) $\dfrac{1}{2}$

(e) ∞

Improper integrals

$$\int_0^1 \frac{2}{x^2 - 1}\, dx =$$

(a) $-\infty$ $\leftarrow$ correct

(b) ∞

(c) $\ln 2$

(d) $-\ln 2$

(e) 0

Sequence

Which of the following is true for the three sequences below?

(I) $a_n = \ln(2n+3) - \ln n$ (II) $a_n = \dfrac{\ln(n^2)}{n}$ (III) $a_n = n \sin\left(\dfrac{1}{n}\right)$

(a) Only (I) and (II) converge.

(b) Only (II) converges.

(c) Only (II) and (III) converge.

(d) Only (I) and (III) converge.

(e) All three converge. $\leftarrow$ correct

Find TRUE statements in the following.

I. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

II. If the nth partial sum, $\{s_n\}$, converges, then $\sum_{n=1}^{\infty} a_n$ converges.

III. The geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ converges if $|r| < 1$.

IV. If $\{a_n\}$ is decreasing and $a_n \geq 0$ for all n, then $\lim_{n\to\infty} a_n = 0$.

(a) I and II only
(b) II and III only $\leftarrow$ correct
(c) III and IV only
(d) I, II, and III only
(e) II, III, and IV only

Remainder of a positive series

11. Let $s = \sum_{n=1}^{\infty} \frac{1}{n^4}$. Using The Remainder Estimate for the Integral Test, find the smallest value of n such that $R_n = s - s_n \leq \frac{1}{81}$.

(a) $n = 2$

(b) $n = 3$

(c) $n = 4$

(d) $n = 5$

(e) $n = 6$

Telescoping series

The series $\sum_{i=1}^{\infty} \left(e^{1/i} - e^{1/(i+1)}\right)$

(a) converges to e

(b) converges to 0

(c) converges to $e - 1$

(d) None of these.

(e) diverges

Remainder of an alternating series

8. The alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+2)^2}$ converges.

Use the Alternating Series Estimation Theorem to determine an upper bound on the absolute value of the error in using s_5 to approximate the sum of the series.

(a) $\frac{1}{8}$

(b) $\frac{1}{9}$

(c) $\frac{1}{64}$

(d) $\frac{1}{81}$

(e) $\frac{1}{35}$

Alternating Series

Which of the following statements is true for the following series?

(I) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (II) $\sum_{n=1}^{\infty} (-1)^n e^{-2n}$ (III) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{(2n+1)!}$

(a) (I) converges, but not absolutely; (II) converges absolutely; (III) diverges.

(b) (I), (II) and (III) converge absolutely.

(c) (I) diverges, (II) converges, but not absolutely; (III) converges absolutely.

(d) (I) converges, but not absolutely; (II) and (III) diverge.

(e) (I) converges, but not absolutely; (II) and (III) converge absolutely.

Ratio Test

Which of the following series is absolutely convergent by **the Ratio Test**?

(I) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{3^n}$ (II) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4(-2)^n}{n!}$ (III) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3+4}$

(a) I and II only $\leftarrow$ correct

(b) I only

(c) II only

(d) II and III only

(e) I, II, and III

Absolute/conditional convergence

(10 points) Determine whether the following series is absolutely convergent, conditionally convergent, or divergent. Show all work, as illustrated in class, by naming the test(s), applying the test(s), and drawing the correct conclusion(s).

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$$

Limit comparison test

19. (8 pts) Determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}+5}{n^3-2n}$ converges or diverges. Support your answer.

MacLaurin Series

4. Find a power series representation for $f(x) = \dfrac{x}{x+4}$ and its radius of convergence.

(a) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{4^{n+1}}$, $R = 4$

(b) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{4^{n+1}}$, $R = \frac{1}{4}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{4^{n+1}}$, $R = 4$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{4^{n+1}}$, $R = \frac{1}{4}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{4^{n+1}}$, $R = 4$

$$f(x) = \ln(x+4)$$

Taylor Series

Find the sum of the series $\sum_{n=2}^{\infty} \frac{3^n}{n!}$.

(a) e^3

(b) $e^3 - 1$

(c) $e^3 - 4$ $\leftarrow$ correct

(d) $e^3 - 5$

(e) ∞

Taylor polynomial

Find a 3rd degree Taylor Polynomial for a function $f(x)$ using the following information.

$$f(1) = 1, \qquad f'(1) = \frac{1}{2}, \qquad f''(1) = -\frac{1}{4}, \qquad f'''(1) = \frac{3}{8}$$

(a) $T(x) = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$

(b) $T(x) = 1 + \frac{1}{2}x - \frac{1}{4}x^2 + \frac{3}{8}x^3$

(c) $T(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

(d) $T(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{3}{8}(x-1)^3$

(e) $T(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$ $\leftarrow$ correct

Radius of convergence

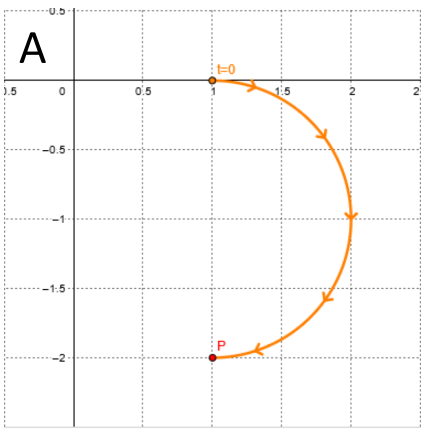
(10 points) Find the radius and interval of convergence of the series. Be sure to test the endpoints of the interval of convergence, and show all work by naming the test(s), applying the test(s), and drawing the correct conclusion(s).

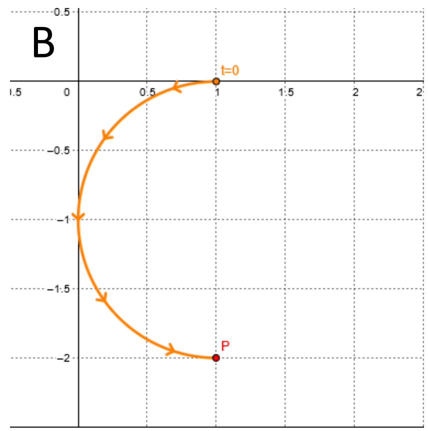
$$\sum_{n=1}^{\infty} \frac{(-1)^n (2x+3)^n}{\sqrt{n}\, 5^n}$$

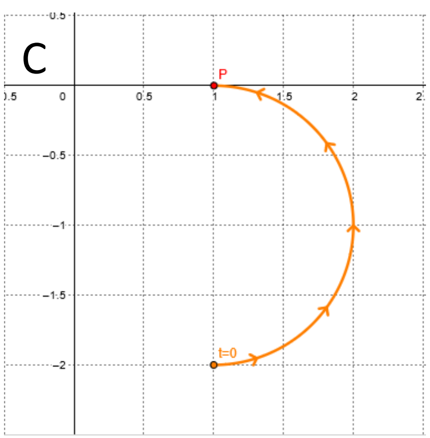
Cartesian eq of a parametric curve

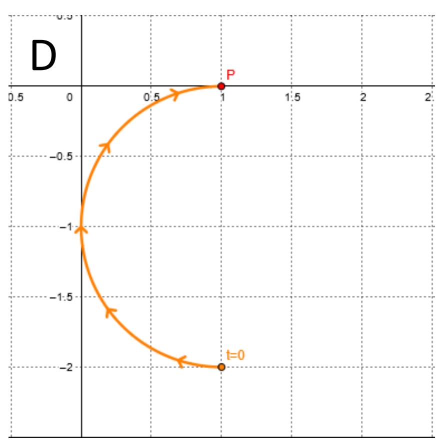
Identify the particle's path by finding a Cartesian equation for it

$x = 1 + \sin t\,, y = \cos t - 1\ (0 \le t \le \pi)$









Length of parametric curves

Find the lengths of the curves$x = \frac{1}{3}(2t+3)^{3/2}, y = t + \frac{t^2}{2}$ $(0 \le t \le 2)$

A. 4
B. 6
C. 8
D. 10
E. None of above

Surface area of the solid by rotating a parametric curve

Find the areas of the surfaces generated by revolving the curves
$x = \cos t, y = 2 + \sin t \ (0 \le t \le 2\pi)$ about x −axis

A. $4\pi^2$
B. $6\pi^2$
C. $8\pi^2$
D. $10\pi^2$
E. None of above

Polar curve graphs

Find the Cartesian equation of $r = 4\tan\theta\sec\theta$

A. $y = 4x$

B. $x = 4y$

C. $y^2 = 4x$

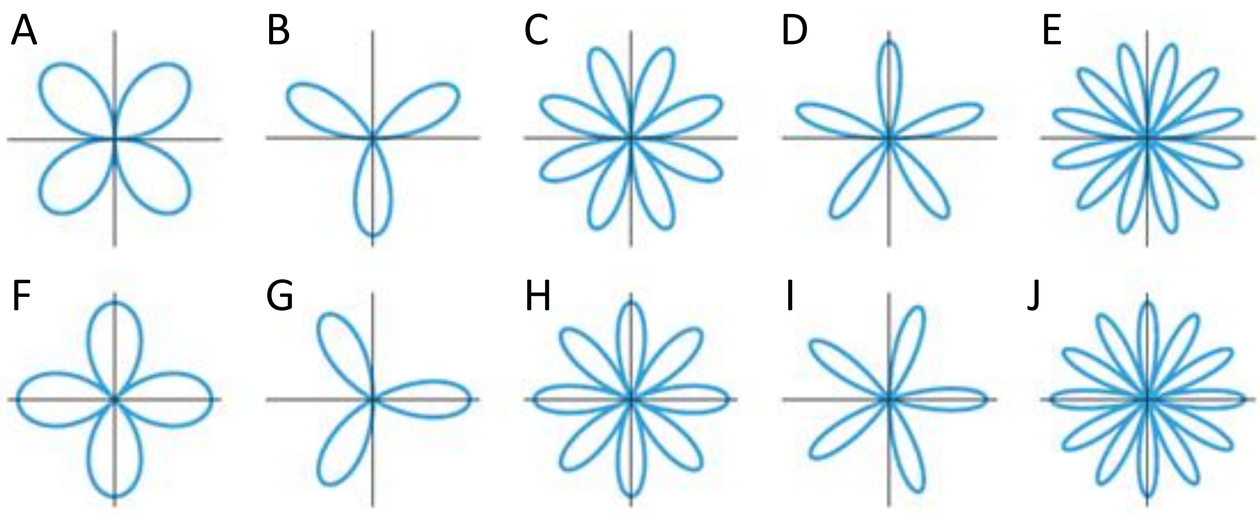
D. $x^2 = 4y$

E. None of above

Cartesian eq of a polar curve

Match the polar equations with the graphs labeled

1. $r = \sin\theta$
2. $r = \sin 2\theta$
3. $r = \sin 3\theta$
4. $r = \sin 4\theta$
5. $r = \sin 5\theta$
6. $r = \sin 6\theta$
7. $r = \cos\theta$
8. $r = \cos 2\theta$
9. $r = \cos 3\theta$
10. $r = \cos 4\theta$
11. $r = \cos 5\theta$
12. $r = \cos 6\theta$



Polar eq of a Cartesian curve

Match the polar equations with the graphs labeled

1. $r = 0.5 + \cos\theta$
2. $r = 1 + \cos\theta$
3. $r = 1.5 + \cos\theta$
4. $r = 2 + \cos\theta$
5. $r = 0.5 + \sin\theta$
6. $r = 1 + \sin\theta$
7. $r = 1.5 + \sin\theta$
8. $r = 2 + \sin\theta$

A B C D