Math 151 - Week-In-Review 8

Topics for the week:

3.9 Related Rates
Review for Exam 2 (3.1 - 3.9)

3.9 Related Rates

1. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal changing when 200 ft of string have been let out?

2. A point moves around the ellipse $4x^2 + 9y^2 = 75$. When the point is at $\left(\sqrt{3}, \sqrt{7}\right)$, its x-coordinate is increasing at a rate of 10 units per second. What is the rate of change of the y-coordinate at that instant?

3. A water tank has the shape of an inverted right circular cone of altitude 18 ft and a base radius of 6 ft. If water is being pumped into the tank at a rate of 10 gal/min(≈ 1.337 ft^3/min), find the rate at which the water level is rising when the water is 5 ft deep.

Review for Exam 2

4. Determine values of x where the function $h(x) = \dfrac{2x}{\sqrt{x^3+4}}$ has a horizontal tangent.

5. Compute the derivative of $f(x) = x^4 \sin(x)$.

6. Compute the derivative of $g(x) = (6 - 3x^2)^5$.

7. Compute the derivative of $h(x) = \arctan\left(x^3 - \sqrt{9-x}\right)$.

8. Compute the derivative of $f(x) = \cos^2\left(12x^{-5}\right)$.

9. Compute the first and second derivatives of $f(x) = \ln(5x^2 - 4)$.

10. Compute the derivative of $h(x) = e^{\log(x^2+1)}$ at $x = 3$.

11. Compute the second derivative $y = \left(\sin^{-1}(2x)\right)^2$.

12. Write the equation of the line tangent to the curve $N(t) = \dfrac{10t}{1 + \frac{t}{10}}$ at $t = 2$.

13. Compute the derivative, $\dfrac{dy}{dx}$ of $4x = \dfrac{3 + y^3}{y^2 + x}$.

14. Determine the unit tangent vector to the curve $\mathbf{r}(t) = (t^2)\mathbf{i} + (3t^3)\mathbf{j}$ at the point where $t = -1$.

15. Assume that $\mathbf{r}(t)$ is a position function for an object. Find the velocity vector(s) and the speed at the point $(3, 0)$ when $\mathbf{r}(t) = \left\langle t^2 - 6t + 8, t^4 - 26t^2 + 25 \right\rangle$.

16. Determine the points on the curve, defined by the parametric equations $x(t) = e^{t^2+4t}$ and $y(t) = 5^{3t+2}$, where the tangent lines are horizontal and where they are vertical.

17. An object is moving in a straight line. The position of the object is given by the equation $s(t) = 4t^3 - 9t^2 + 6t + 2$, where t is measured in seconds and s is measured in meters.

    a. Compute the velocity and acceleration of the object at time t.

    b. When is the object at rest?

18. Suppose that the population size of a bacteria at time t days is $N(t) = N_0 7^{0.05t}$, $t \geq 0$ where $N_0 = 1200$ is the initial population. Compute the rate of growth of the bacteria population after 6 days.

19. When an energy drink is removed from the refrigerator, its temperature is 35°F. After 30 minutes, in a room registering 72°F, the energy drink temperature has increased to 60°F.

    a. What is the temperature of the drink after 45 minutes?

    b. At what time does the energy drink reach a temperature of 70°F?

20. Compute the 26th derivative of $g(x) = x\sin(x)$.

21. Compute the third derivative of $h(x) = e^{-x^2+1}$.