6.2: SOLVING ODES WITH LAPLACE TRANSFORMS

Review

- Laplace transform of derivatives
  - $\mathcal{L}\{f'\} =$
  - $\mathcal{L}\{f''\} =$
  - $\mathcal{L}\{f'''\} =$
- How to solve differential equations with the Laplace transform
  - Laplace transform
  - Solve for $Y(s)$
  - Inverse transform

Exercise 1

Solve the initial value problem

$$y'' - 5y' + 6y = 0, \quad y(0) = -1, \quad y'(0) = 0.$$

Exercise 2

Solve the initial value problem

$$y'' + 3y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 2.$$

Exercise 3

Solve the initial value problem

$$y'' + 4y' + 4y = 2e^{t}, \quad y(0) = 0, \quad y'(0) = 3.$$

Exercise 4

Solve the initial value problem

$$y''' + y' = 0, \quad y(0) = 0, \quad y'(0) = 4, \quad y''(0) = 2.$$

6.3: STEP FUNCTIONS

Review

- The **unit step function** u_c is defined by

- It can be used to write discontinuous functions into a single equation.
- The Laplace transform of u_c is

- Laplace transforms of **shifts**

- Inverse Laplace transform of **shifts**

Exercise 5

Convert the following function to a piecewise function. Also, graph the function. Compute its Laplace transform.

$$f(t) = u_2(t) - 4u_3(t)$$

Exercise 6

Convert the following function to a piecewise function. Compute its Laplace transform.

$$f(t) = t^2 - \sin(t-1)u_1(t) - t^2u_2(t)$$

Exercise 7

Convert the following piecewise function into a form that involves step functions.

$$g(t) = \begin{cases} 0 & t < 3 \\ 5 & 3 \leq t < 6 \\ \cos(2t) & t \geq 6 \end{cases}$$

Exercise 8

Convert the following piecewise function into a form that involves step functions. Compute it's Laplace transform.

$$g(t) = \begin{cases} 2t & t < 4 \\ e^{2t} & t \geq 4 \end{cases}$$

6.4: DIFFERENTIAL EQUATIONS WITH DISCONTINUOUS FORCING FUNCTIONS

Exercise 9

Solve the initial value problem.

$$f'' + f = u_2(t), \quad f(0) = 0, \quad f'(0) = 0.$$

Exercise 10

Solve the initial value problem.

$$w'' + w' = \begin{cases} 2 & t < 4 \\ 0 & t \geq 4 \end{cases}, \quad w(0) = 0, \quad w'(0) = 0.$$

Exercise 11

Consider a spring and mass system with a 4 kg mass hanging on a spring. When the mass is hung on the spring, the spring extends 40 cm. The mass experiences a damping force of 6 N when the mass is moving 3 m/s. The mass starts from equilibrium at rest, but there is an external force $\sin(t)$ that lasts for the first 2π seconds. Write down the initial value problem that describes this situation.