03

MATH 152
Week in Review

6.4 Work

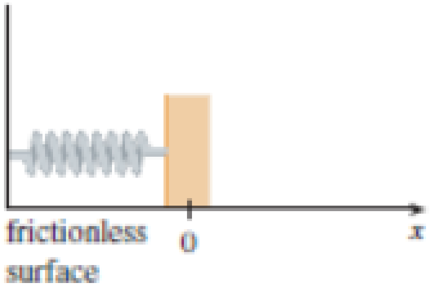
7.1 Integration by Parts

MATH 152 | Week in Review

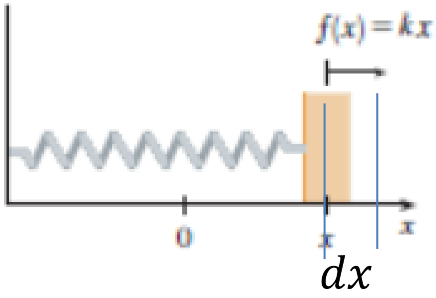
- **Weight = Force**
- **Work = Force*Distance**

  = Weight*Distance

Spring problem (Hooke's law & spring constant)



(a) Natural position of spring



$$dW = F(x)dx$$

Work done by stretching a spring by d_1 to d_2 where the resting length is d_0. (Recall Hooke's law: $F = kx$)

- **Step 1** : plot a graph in the coordinate system
  - Set the the resting length $= 0$
    - $\begin{cases} d_0 \Rightarrow x = 0 \\ d_1 \Rightarrow x_1 = d_1 - d_0 \\ d_2 \Rightarrow x_2 = d_2 - d_0 \end{cases}$
- **Step 2:** Slicing the spring by dx segment and consider a segment at location x (to be stretched by dx)
  - Assume the force over $[x, x + dx]$ is constant, $F(x)$
- **Step 3:** Find the work done by $F(x)$ over $[x, x + dx]$

  $$dW = F(x)dx = (kx)dx$$
- **Step 4.** Find the total work done by stretching the spring over $x \in [x_1, x_2]$ by integrating dW
  - $W = \int dW$

    $$= \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2} (kx)dx$$

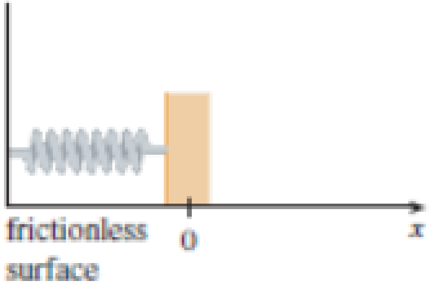
    $$= \frac{k}{2}[x^2]_{x_1}^{x_2} = \frac{k}{2}(x_2^2 - x_1^2) = \frac{k}{2}(x_2 + x_1)(x_2 - x_1)$$

Step 4. Find the spring constant

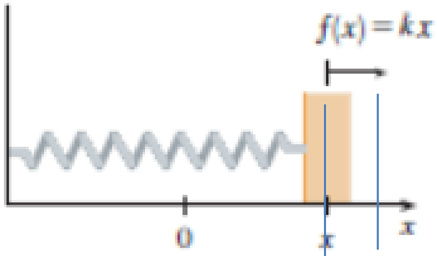
- $k = \frac{2W}{(x_2+x_1)(x_2-x_1)}$

An ideal spring has a natural length of 10 meters. The work done in stretching the spring from 14 meters to 18 meters is 24J. Determine the spring constant k.

(a) $k = \frac{1}{2}\,\mathrm{N/m}$
(b) $k = \frac{3}{8}\,\mathrm{N/m}$
(c) $k = 1\,\mathrm{N/m}$
(d) $k = 3\,\mathrm{N/m}$
(e) $k = 6\,\mathrm{N/m}$



(a) Natural position of spring

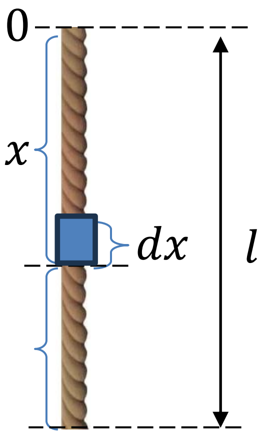


dx

$dW = F(x)dx$

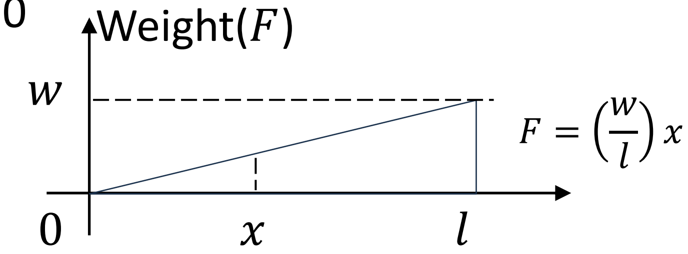
- Hooke's law: $F(x) = kx$ w/ x =displacement
- Adjust for the resting position: $10 \Rightarrow\ x = 0$
  - $\begin{cases} 10 \Rightarrow x = 0 \\ 14 \Rightarrow x = 4 \\ 18 \Rightarrow x = 8 \end{cases}$
- Work needed to stretch from x to $x + dx$
  - $dW = F(x)dx$
    $= (kx)dx$
- Work needed to stretch from 4 to 8
  - $W(24) = \int dW = \int F(x)dx$
    $= \int_4^8 (kx)dx$
    $= \left[\frac{1}{2}kx^2\right]_4^8 = k\frac{8^2-4^2}{2}$
    $= k\frac{(8-4)(8+4)}{2}$
    $= k\frac{4\cdot 12}{2} = 24k$
  - $24 = 24k\ \Rightarrow\ k = 1$

Lifting problem



Work done by lifting a cable weighing w lb with a length of l fts.

- **Step 1** : plot a graph in the coordinate system (**weight vs length**)
  - Set the top of the rope $= 0$

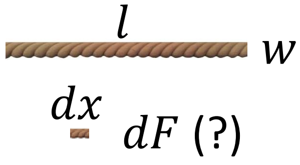


- **Step 2:** Slicing the cable by dx segment and consider a segment at location x (to be lifted by x)
  - Find the weight of rope with length x (=force, F)
    - $F = \frac{w}{l}x$
- **Step 3:** Find the weight of dx cable segment (2 different ways)
  - Let dF =Weight of dx cable segment:
    - By differentiating $F = \frac{w}{l}x$,

$$dF = \frac{w_0}{l}dx$$

- **Step 4.** Find the work done by lifting a cable segment dF lb with a length of $x$fts.
  - $dW = (dF)x = \left[\frac{w}{l}dx\right]x = \frac{w}{l}xdx$
- **Step 5.** Find the total work by integrating dW
  - $W = \int_0^l \frac{w}{l}xdx$

Another way to find the weight of dx rope segment



$$\frac{dF}{dx} = \frac{w}{l}$$

$$dF = \frac{w_0}{l}dx$$

12. A 90 ft cable weighing 10 lb is hanging down the side of a 200 ft building. How much work is required to pull the rope 30 feet up the side of the building?

(a) 6000 ft-lb

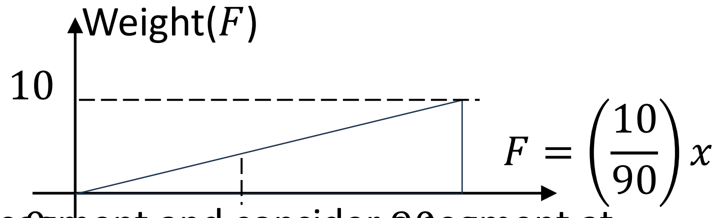
(b) 1500 ft-lb

(c) 250 ft-lb

(d) 300 ft-lb

(e) 50 ft-lb

- **Step 1** : plot a graph in the coordinate system (**weight vs length**)
  - Set the top of the rope = 0

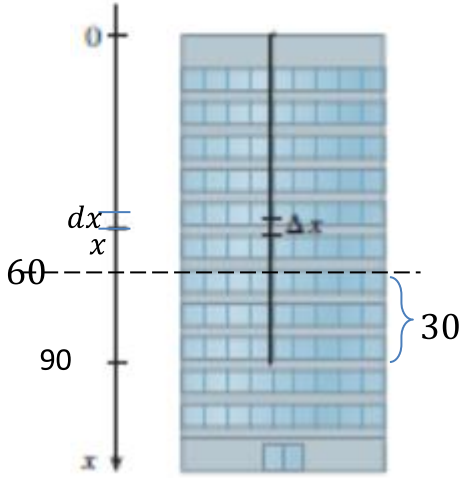


- **Step 2:** Slicing the cable by dx segment and consider segment at location x (to be lifted by x)
  - Find the weight of rope with length x (=force, F)
    - $F = \frac{10}{90}x$
- **Step 3:** Find the weight of dx cable segment
  - Let dF =Weight of dx cable segment:
    - By differentiating $F = \frac{10}{90}x$,

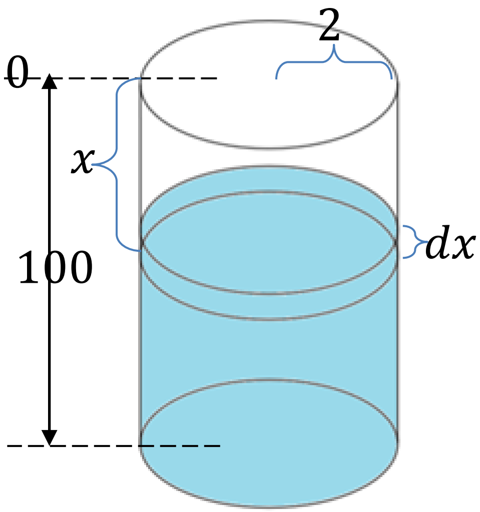
$$dF = \frac{10}{90}dx$$

- **Step 4.** Find the work done by lifting a cable segment dF lb with a length of $x$fts.
  - $dW = (dF)x = \left[\frac{10}{90}dx\right]x = \frac{1}{9}xdx$
- **Step 5.** Find the total work by integrating dW
  - $W = \int_{60}^{60} \frac{1}{9}xdx = \frac{1}{2} \cdot \frac{1}{9}[x^2]_{60}^{90}$

$$= \frac{1}{2} \cdot \frac{1}{9}[90^2 - 60^2] = \frac{(90-60)(90+60)}{2\cdot 9} = \frac{9(30-20)(30+20)}{2\cdot 9} = \frac{9\cdot 500}{2\cdot 9} = 450$$

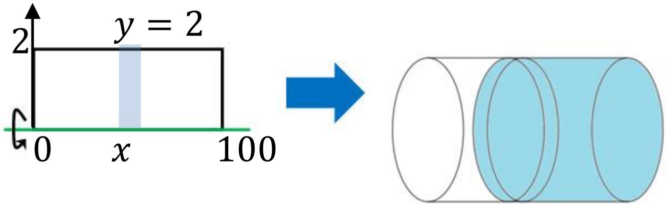


Lifting problem overview



A 200-lb liquid is inside of 100 ft long cylinder with radius 2ft. How much work is required pump the water out of the cylinder? Assume the weight density of the liquid = $2\ \mathrm{lb/ft^3}$

- **Step 1** : plot a graph in the coordinate system (tank shape vs depth): Set the top of the tank= 0



- **Step 2:** Slicing the cylinder by dx height (Set the top $= 0$) and consider a disc at location x (to be lifted by x)
  - Find the volume of the disc at x
    - $dv = (\pi 2^2)dx = 4\pi dx$
- **Step 3:** Find the weight of water within the disc (=force, F)
  - **water weight = (water volume)x(weight density)**
    - $dF = \rho(\pi r^2)dx = 2(4\pi dx) = 8\pi dx$
- **Step 4.** Find the work done by pumping the water disc dF lb by a length of $x$fts.
  - $dW = (dF)x = [8\pi dx]x = 8\pi x dx$
- **Step 5.** Find the total work by integrating dW
  - $W = \int_0^{100} 8\pi x dx dx$

Another way to find the dF (weight of a disc)

Step1. Find the weight up to depth x

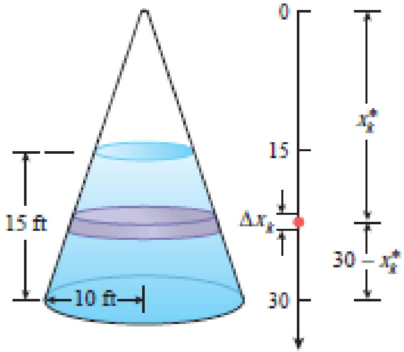
$$W = \rho \int_0^x (\pi r^2)dx$$
$$= \rho[\pi(r^3/3)]$$

Step2. Differentiate W

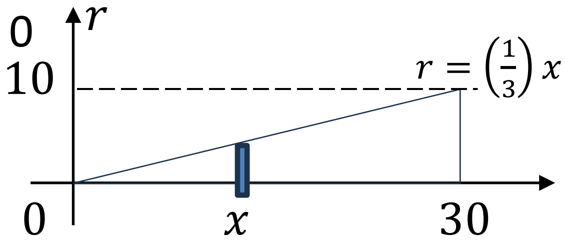
$$dW = \rho(\pi r^2)dx$$

Example (water level = integration limit)

A conical container has the radius 10 ft and height 30 ft. Suppose that this container is filled with water to a depth of 15 ft. How much work is required to pump all of the water out through a hole in the top of the container? Use the weight density of water = $62.4\ \mathrm{lb/ft^3}$

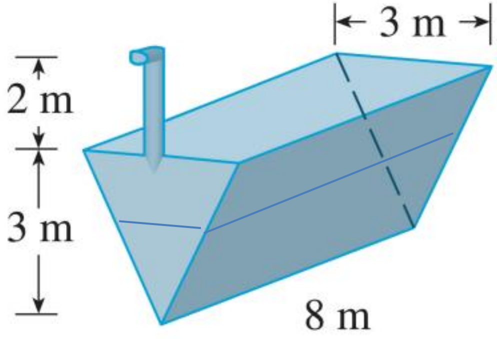


- **Step 1** : plot a graph in the coordinate system (tank shape vs depth): Set the top of the tank $= 0$



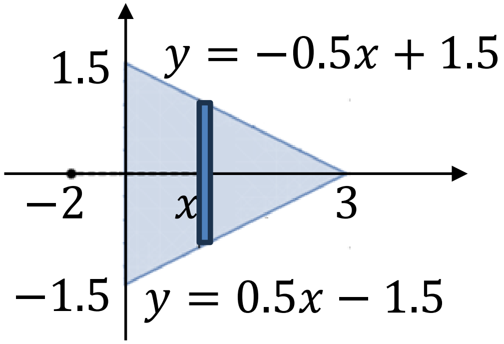
- **Step 2:** Slicing the cylinder by dx height (Set the top $= 0$) and consider a disc at location x (to be lifted by x)
  - Find the volume of the disc at x
    - $dv = (\pi r^2)dx = \pi(x^2/9)dx$
- **Step 3:** Find the weight of water within the disc (=force, F)
  - **water weight = (water volume)x(weight density)**
    - $dF = \rho(\pi r^2)dx = 62.4\pi(x^2/9)dx = 6.93\pi x^2 dx$
- **Step 4.** Find the work done by pumping the water disc dF lb by a length of $x$fts.
  - $dW = (dF)x = [6.93\pi x^2 dx]x = 6.93\pi x^3 dx$
- **Step 5.** Find the total work by integrating dW (Limit ??)
  - $W = \int_{\mathbf{15}}^{30} 6.93\pi x^3 dx$

Example (spout = moving distance)

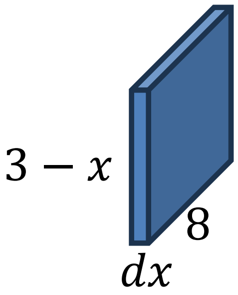


The tank shown is full of water. Find the work required to pump the water out of the spout. (Use 9800 N/m^3 as water density)

- **Step 1** : plot a graph in the coordinate system (tank shape vs depth): Set the top of the tank $= 0$



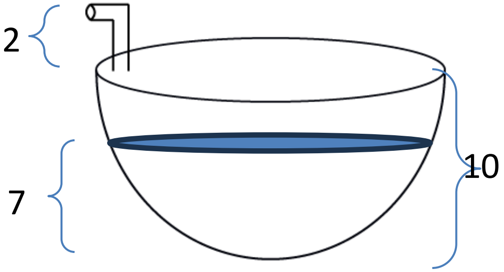
- **Step 2:** Slicing the tank by dx height (Set the top $= 0$) and consider a slab at location x (to be lifted by x)
  - Find the volume of the disc at x
    - $dv = 8(3 - x)dx$
- **Step 3:** Find the weight of water within the disc (=force, F)
  - **water weight = (water volume)x(weight density)**
    - $dF = \rho dv = 9800 \cdot [8(3 - x)dx]$
- **Step 4.** Find the work done by pumping the water disc dF lb by a length of $x + 2$fts (due to spout).
  - $dW = (dF)x = \left[9800 \cdot [8(3 - x)dx]\right](x + 2)$
    $= 9800 \cdot [8(3 - x)(x + 2)\ ]dx$
- **Step 5.** Find the total work by integrating dW (Limit ??)
  - $W = (8 \cdot 9800) \int_0^3 (3 - x)(x + 2)dx$

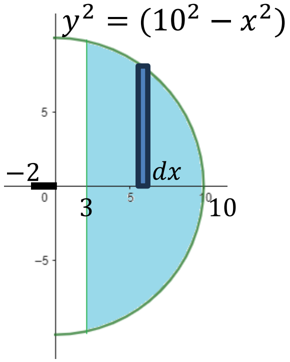


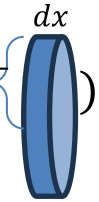
$(-0.5x + 1.5) - (0.5x - 1.5)$
$= 3 - x$

3.2 Integration by Parts

A hemispherical tank has the shape shown below. The tank has a radius of 10 meters with a 2 meter spout at the top of the tank. The tank is filled with water to a depth of 7 meters. The weight density of water is $\rho = 9800 N/m^3$. Suppose we want to find the work required to pump the water through the spout







$$V(\sqrt{10^2 - x^2}) = \pi r^2 dx$$

$$= \pi(10^2 - x^2)dx$$

The tank shown is full of water. Find the work required to pump the water out of the spout. (Use 9800 N/m^3 as water density)

- **Step 1** : plot a graph in the coordinate system (tank shape vs depth): Set the top of the tank $= 0$
- **Step 2:** Slicing the tank by dx height (Set the top $= 0$) and consider a slab at location x (to be lifted by x)
  - Find the volume of the disc at x
    - $dv = \pi(10^2 - x^2)dx$
- **Step 3:** Find the weight of water within the disc (=force, F)
  - **water weight = (water volume)x(weight density)**
    - $dF = \rho dv = 9800\pi(10^2 - x^2)dx$
- **Step 4.** Find the work done by pumping the water disc dF lb by a length of x + 2fts (due to spout).
  - $dW = (dF)x = [9800\pi(10^2 - x^2)dx](x + 2)$
    $= 9800\pi(10^2 - x^2)(x + 2)dx$
- **Step 5.** Find the total work by integrating dW (Limit ??)
  - $W = 9800\pi \int_3^{10} (10^2 - x^2)(x + 2)dx$

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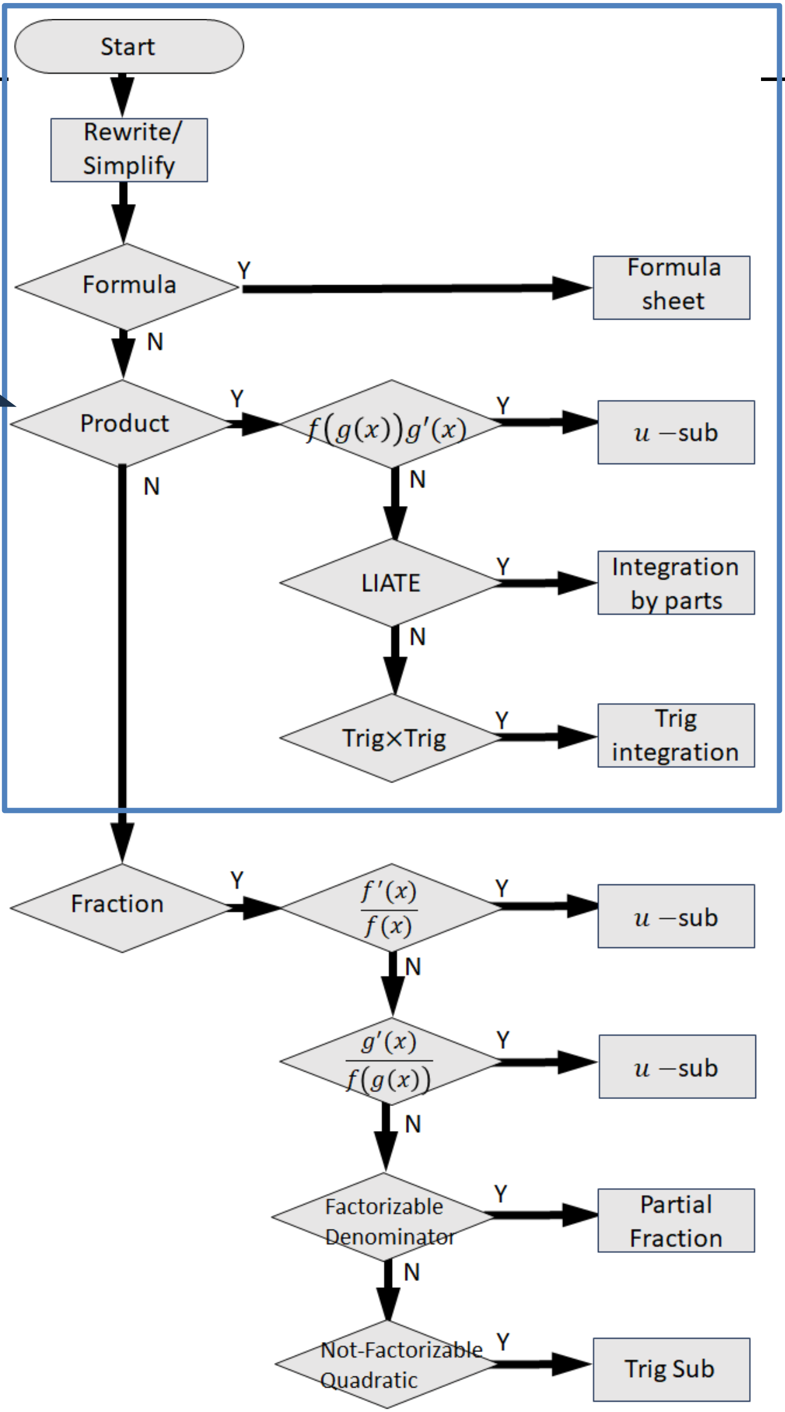
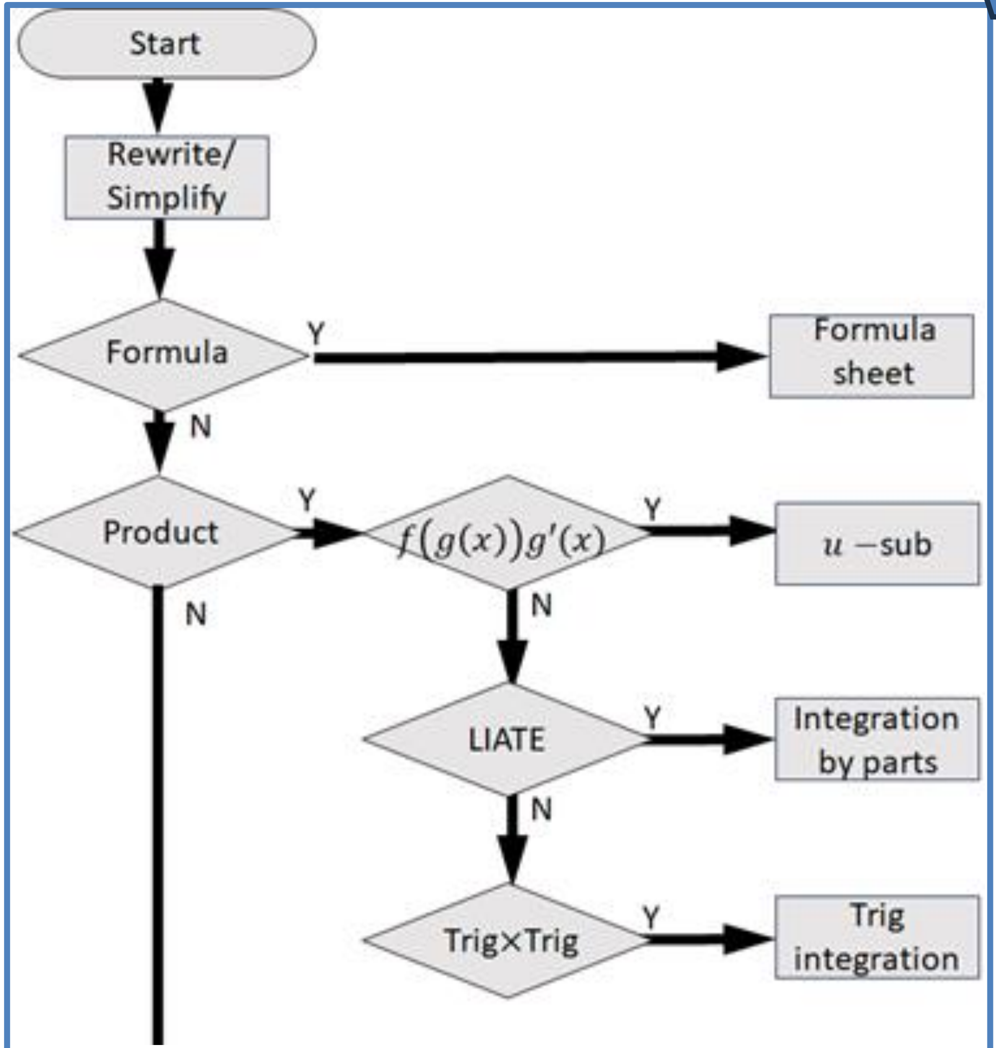




Choice of u and v: LIATE method for IBP $\int uv'\,dx = uv - \int u'v\,dx$

- The LIATE method offers guidelines for determining when and how to apply IBP
  - IBP may work for uv' when u and v' are **LIATE** functions
  - u and v' can be chosen by **LIATE** order
- **LIATE method** (general guideline for IBP: combinations of LIATE ⇒ IBP)

u----**L**ogarithmic, **I**nverse trigonometric, **A**lgebraic, **T**rigonometric, **E**xponential----v'

- Why LIATE works for IBP: $\int uv'\,dx = uv - \int u'v\,dx$?
  - u becomes "simpler" when differentiated ($\int u'v\,dx$ becomes easier).
  - v' is readily integrated to obtain v.
- *For IBP, we need* u, u' *and* v, v'

Example Evaluate $\int xe^x dx$

u----**L**ogarithmic, **I**nverse trigonometric, **A**lgebraic, **T**rigonometric, **E**xponential----v'

x $\qquad$ e^x

u'-- 1 $\qquad$ e^x--v

$$\underset{\int uv'dx =}{\int xe^x dx =} \quad \underset{uv}{xe^x} \quad \underset{-\int u'vdx}{-\int 1\cdot e^x dx}$$

$$= xe^x - e^x + C$$

Tabular method

Evaluate $\int \ln x \, dx$

[Hint: Consider $\int 1 \cdot \ln x \, dx$

u----**L** **I** **A** **T** **E**----v'

$\ln x \quad 1$

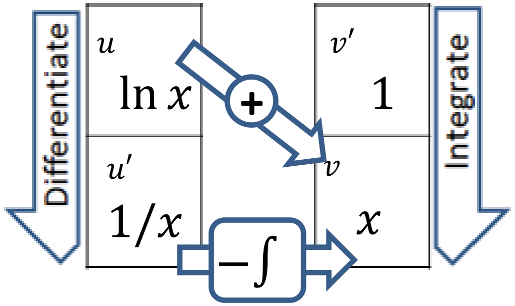
u'-- $\frac{1}{x} \quad x$ --v

$$\int \ln x \times 1dx = \quad x\ln x \quad - \int \frac{1}{x} \cdot xdx$$

$$\int uv' = \quad uv \quad - \int u'v$$

$$= x\ln x - \int 1dx$$

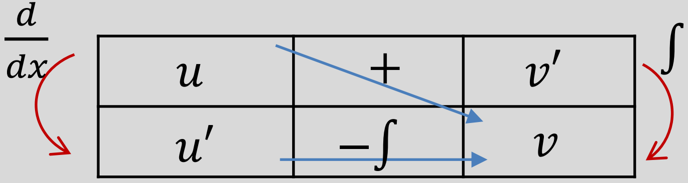
$$= x\ln x - x + C$$



Formula method

$$\int uv' = \quad uv \quad -\int u'v$$

Tabular method



u	$+$	v'
u'	$-\int$	v

Antiderivative of $\ln x$

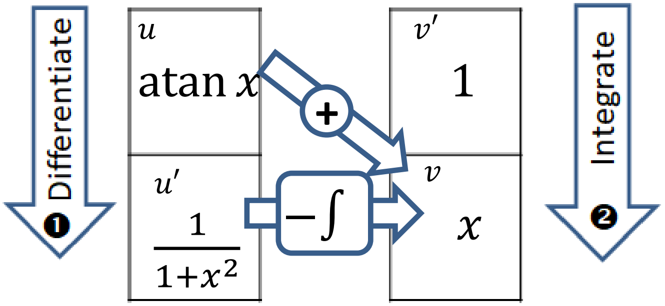
$$\int \ln x \, dx = x\ln x - x + C$$

Example

Compute $\int_0^1 \arctan x \, dx$.

(a) $\frac{\pi}{4} - \frac{1}{2}\ln 2$
(b) $\frac{\pi}{4} - \ln 2$
(c) $1 - \frac{1}{2}\ln 2$
(d) $1 - \ln 2$
(e) $\frac{\pi}{4}$

Evaluate $\int \tan^{-1} x \, dx$ by the tabular method
Hint: $\int \tan^{-1} x \, dx = \int 1 \cdot \tan^{-1} x \, dx$



u----**L I A T E**----v'
$\tan^{-1} x \quad 1$

$$\int_0^1 \tan^{-1} x = [x \tan^{-1} x]_0^1 \quad - \int_0^1 \frac{x}{1+x^2} dx$$

$$= [x \tan^{-1} x]_0^1 - \frac{1}{2}[\ln(1+x^2)]_0^1 \text{ (}u\text{-sub)}$$

$$= (\tan^{-1}(1) - 0) - \frac{1}{2}(\ln 2 - \ln 1)$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln 2$$

Repeated IBP

Vanishing Repeated IBP case

- $\int x^n e^x dx$ or $\int [\text{Polynomials}] e^x dx$
- $\int x^n \text{sin} x dx$ or $\int [\text{Polynomials}] \sin x \, dx$
- $\int x^n \text{cos} x dx$ or $\int [\text{Polynomials}] \cos x \, dx$

Repeat IBP until polynomials vanish by differentiation

Cyclic Repeated IBP case

- $\int e^x \text{sin} x dx$
- $\int e^x \text{cos} x dx$

Repeat IBP until the same trigonometric function appears by differentiation

- $\sin x \underset{\frac{d}{dx}}{\Rightarrow} \cos x \underset{\frac{d}{dx}}{\Rightarrow} -\sin x$
- $\cos x \Rightarrow -\sin x \Rightarrow -\cos x$

Vanishing Repeated IBP ($\int x^n e^x dx$ or $\int x^n \sin x dx$)

Evaluate $\int x^2 e^x\,dx$

u----L I A T E----v'

$x^2 \quad e^x$

$2x \quad e^x$

$2 \quad e^x$

$$\underset{\int uv' =}{\int \mathbf{x^2e^x dx} =} \quad \underset{uv}{x^2e^x} \quad \underset{-\int u'v}{-\int 2xe^x dx}$$

$$= x^2e^x - \underset{-\int uv' =}{\int \mathbf{2xe^x dx}} \text{ (Another IBP)}$$

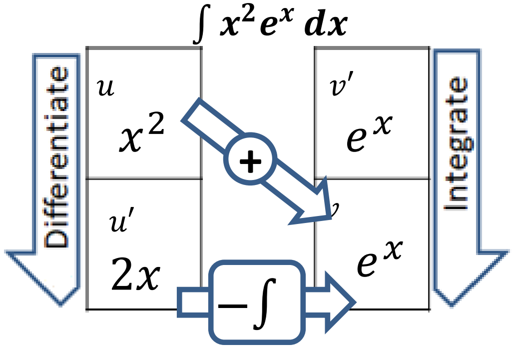
$$= x^2e^x \quad \underset{-uv}{-2xe^x} \quad \underset{+\int u'v}{+\int 2e^x\,dx}$$

$$= x^2e^x - 2xe^x + \underset{\int uv' =}{\int \mathbf{2e^x dx}} \text{ (Another IBP)}$$

$$= x^2e^x - 2xe^x \quad \underset{+uv}{+2e^x} \quad \underset{-\int u'v}{-\int 0 \cdot e^x dx}$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

We can combine these steps into one table



$\int \mathbf{x^2e^x\,dx}$

Differentiate

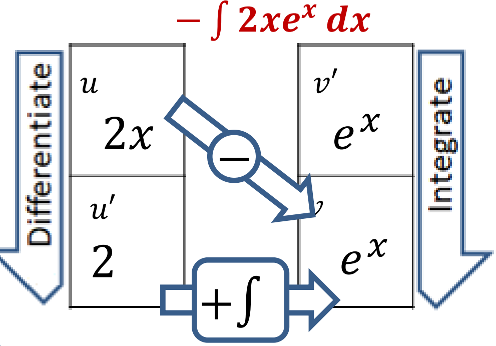
Integrate

u x^2		v' e^x
u' $2x$		v e^x

+

$-\int$

Because of the $-$ sign in $\int \mathbf{2xe^x dx}$, we switch the sign associated with multiplication



$-\int \mathbf{2xe^x\,dx}$

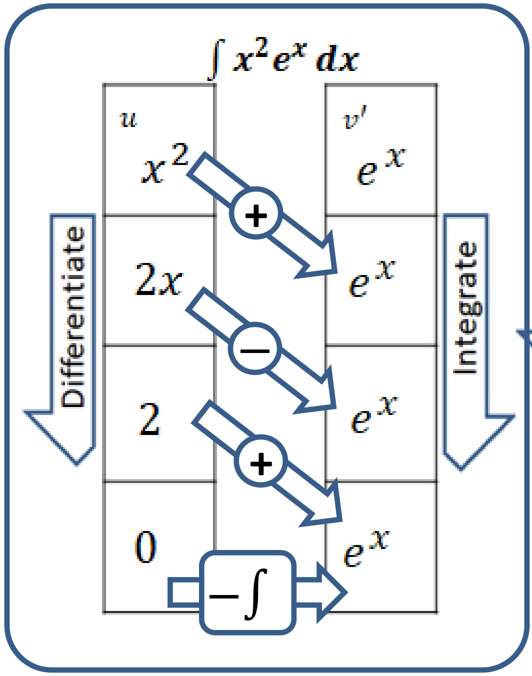
Differentiate

Integrate

u $2x$		v' e^x
u' 2		v e^x

$-$

$+\int$



$\int \mathbf{x^2e^x\,dx}$

Differentiate

Integrate

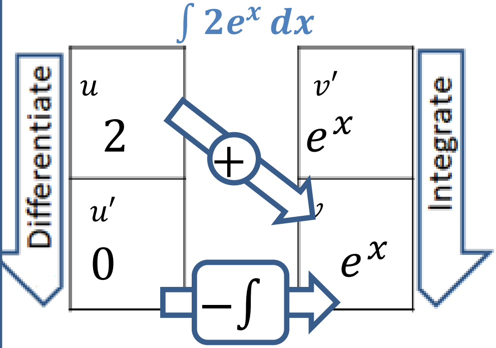
u x^2		v' e^x
$2x$		e^x
2		e^x
0		e^x

+

$-$

+

$-\int$



$\int \mathbf{2e^x\,dx}$

Differentiate

Integrate

u 2		v' e^x
u' 0		v e^x

+

$-\int$

Example

Evaluate $\displaystyle\int_0^1 \frac{x^2}{e^x}\,dx$.

(a) $2 - \dfrac{5}{e}$

(b) $\dfrac{5}{e} - 2$

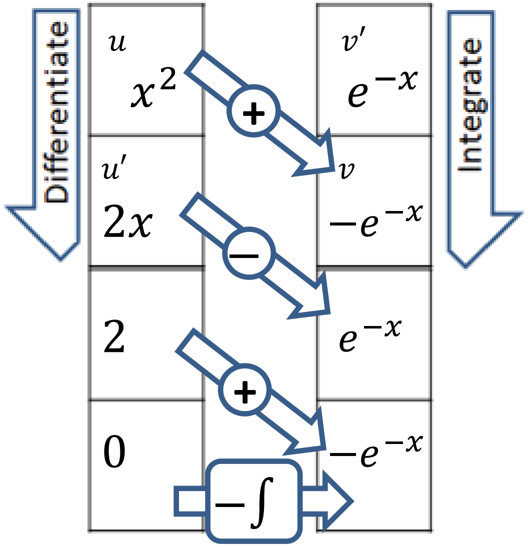
(c) $1 - \dfrac{3}{e}$

(d) $1 - \dfrac{2}{e}$

(e) $1 - \dfrac{1}{e}$

u----**L I A T E**----v'

$x^2 \quad e^{-x}$



$$\int_0^1 \frac{x^2}{e^2}\,dx = \left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^1$$
$$= (-e^{-1} - 2e^{-1} - 2e^{-1}) - (-2)$$
$$= 2 - 5e^{-1}$$

Example

Compute $\int x \ln x \, dx$

[Sol 1] IBP u----**L I A T E**----v'

$\ln x \quad x$

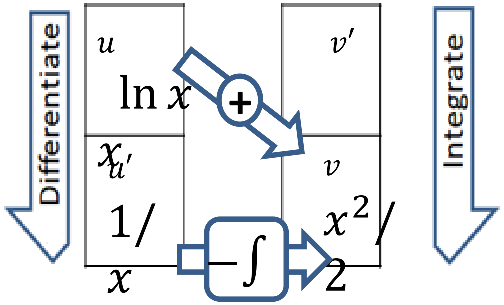
u'- $\frac{1}{x} \quad \frac{x^2}{2}$ -v

$$\int \ln x \cdot x dx = \frac{x^2}{2} \ln x \quad - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$\int uv' = \qquad uv \qquad - \int u'v$$

$$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$



[Sol2] u −sub

$$\int x \ln x \, dx = \frac{1}{2} \int x (2 \ln x) dx = \frac{1}{2} \int x \ln x^2 \, dx$$

u −sub : $u = x^2 \quad \Rightarrow du = 2x dx$

$$= \frac{1}{4} \int \ln u \, du$$

$$= \frac{1}{4} [u \ln |u| - u] + C$$

$$= \frac{1}{4} [x^2 \ln x^2 - x^2] + C$$

Cyclic Repeated IBP ($\int e^x \sin x dx$ or $\int x^n \cos x dx$)

Evaluate $\int e^x \sin x \, dx$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x + \int e^x(-\sin x)dx$$

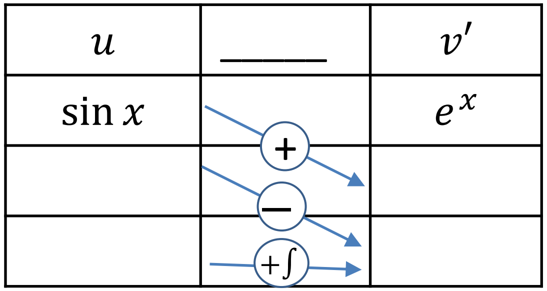
$$\boldsymbol{\int e^x \sin x \, dx} = e^x \sin x - e^x \cos x - \boldsymbol{\int e^x \sin x \, dx}$$

$$2\boldsymbol{\int e^x \sin x \, dx} = e^x(\sin x - \cos x)$$

$$\boldsymbol{\int e^x \sin x \, dx} = \frac{1}{2}e^x(\sin x - \cos x)$$

u----L I A T E----v'

$\sin x$ e^x



u	______	v'
$\sin x$	$+$	e^x
	$-$	
	$+\int$	

Evaluate $\int e^x \cos x \, dx$

$$\int e^x \cos x \, dx = e^x \cos x - e^x(-\sin x) + \int e^x(-\cos x)dx$$

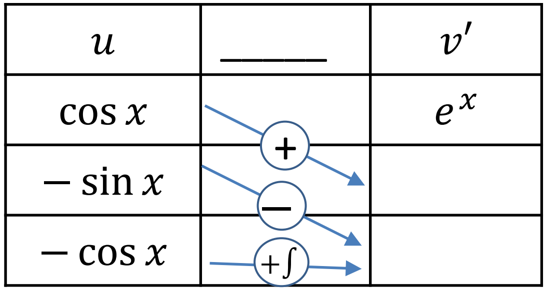
$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$2\int e^x \cos x \, dx = e^x(\sin x + \cos x)$$

$$\int e^x \cos x \, dx = \frac{1}{2}e^x(\sin x + \cos x)$$

u----L I A T E----v'

$\cos x$ e^x



u	______	v'
$\cos x$	$+$	e^x
$-\sin x$	$-$	
$-\cos x$	$+\int$	

Example (u −sub + IBP)

Compute $\int x^5 e^{x^3}\,dx$

Rewrite

$$\int x^5 e^{x^3} dx = \int x^3 e^{x^3} (x^2 dx)$$

u −sub

$$u = x^3 \;\Rightarrow du = 3x^2 dx \quad \Rightarrow x^2 dx = \frac{1}{3} du$$

$$\int x^3 e^{x^3} (x^2 dx) = \int u e^u \left(\frac{1}{3} du\right)$$

$$= \frac{1}{3} \int u e^u du$$

$$= \frac{1}{3} [u e^u - e^u] + C$$

$$= \frac{1}{3} x^3 e^{x^3} - e^{x^3} + C$$

Evaluate $\int x e^x dx$

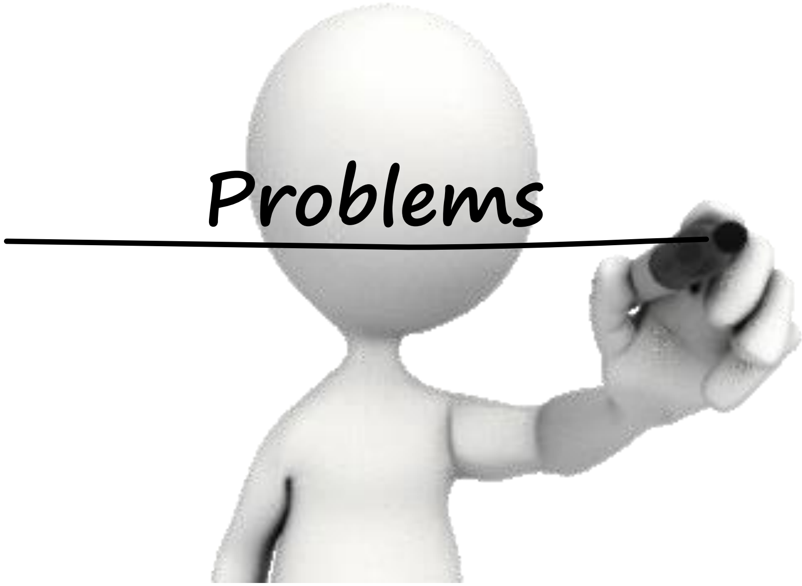
u----**L**ogarithmic, **I**nverse trigonometric, **A**lgebraic, **T**rigonometric, **E**xponential----v'

x $\quad$ e^x

u'-- 1 $\quad$ e^x--v

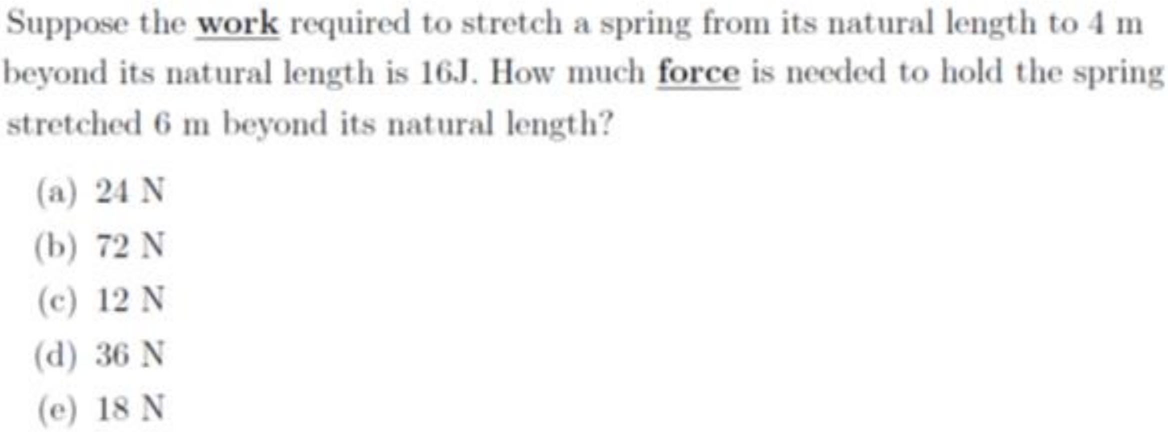
$$\underbrace{\int x e^x dx =}_{\int u v' dx =} \underbrace{x e^x}_{uv} \underbrace{- \int 1 \cdot e^x dx}_{- \int u' v dx}$$

$$= x e^x - e^x + C$$



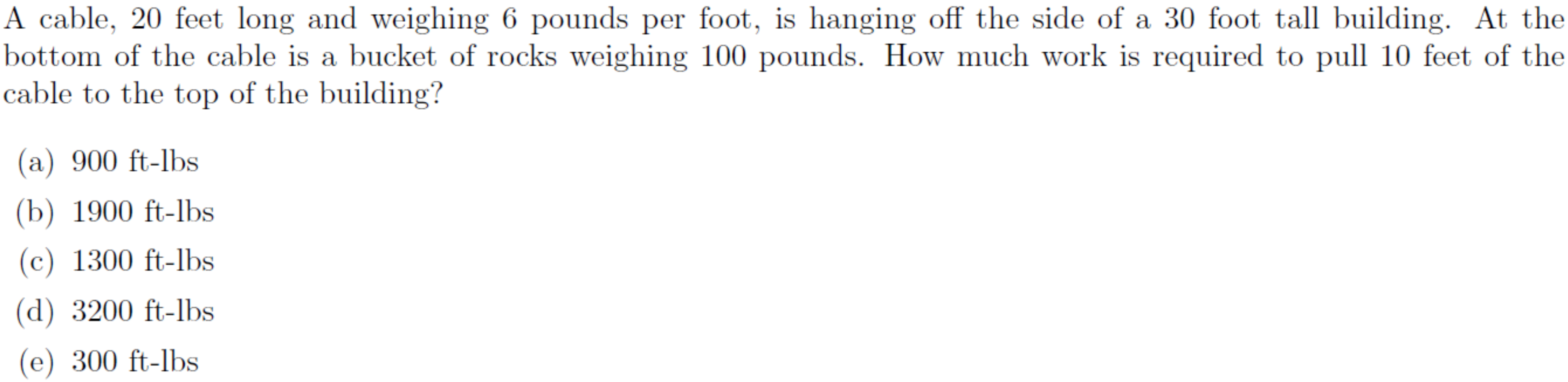
Problems

Exercise



Suppose the **work** required to stretch a spring from its natural length to 4 m beyond its natural length is 16J. How much **force** is needed to hold the spring stretched 6 m beyond its natural length?

(a) 24 N
(b) 72 N
(c) 12 N
(d) 36 N
(e) 18 N



A cable, 20 feet long and weighing 6 pounds per foot, is hanging off the side of a 30 foot tall building. At the bottom of the cable is a bucket of rocks weighing 100 pounds. How much work is required to pull 10 feet of the cable to the top of the building?

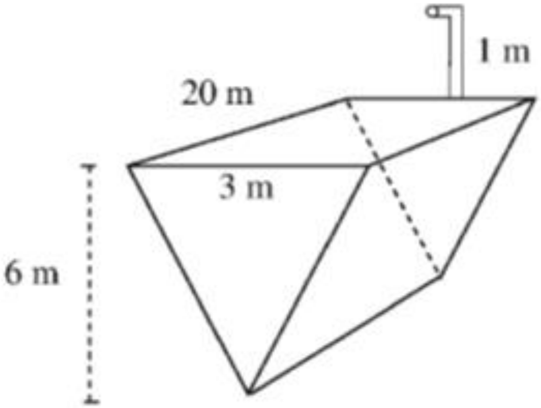
(a) 900 ft-lbs
(b) 1900 ft-lbs
(c) 1300 ft-lbs
(d) 3200 ft-lbs
(e) 300 ft-lbs

Exercise

A tank filled with water is in the shape of a trough with isosceles triangles at its ends. The trough is 20 meters long, has a height of 6 meters, and the width of the trough across the top is 3 meters. The trough has a spout with height 1 meter. The weight density of water is $\rho g = 9800 N/m^3$.

Set up an integral that will compute the work required to pump all the water out of the spout. **Do not evaluate**

Clearly indicate on the picture where you are placing the axis and which direction is positive.



A tank is in the shape of an inverted cone with radius $r = 4$ feet and height $h = 20$ feet. Assuming it is full of water, set up but **do not evaluate** an integral that gives the work it takes to pump the water through a 1 foot tall spout located at the top of the tank. Use $\rho g = 62.5$ pounds per cubic foot. **Clearly indicate where you are placing the axis and which direction is positive.**

Exercise

A chain 30 meters long and weighing 24 newtons per meter hangs from the top of a 50 meter tall building. Calculate the work done in pulling the first 10 meters of this chain to the top of the building.

(a) 1200 Joules
(b) 3000 Joules
(c) 9000 Joules
(d) 6000 Joules
(e) None of the above

A force of 40 N is required to hold a spring that has been stretched from its natural length of 1 m to a length of 3 m. Find the work required to stretch this spring from a length of 4 meters to a length of 5 meters.

(a) 90 Joules
(b) 70 Joules
(c) 80 Joules
(d) 100 Joules
(e) None of the above

A 20 ft rope weighing 0.1 lb/ft is hanging down the side of a 20 ft building. There is a 5 lb bucket attached to the rope. How much work is required to pull the rope with the bucket 2 ft up the side of the building?

(a) 6.8 $ft-lb$
(b) 13.8 $ft-lb$
(c) 14 $ft-lb$
(d) 14.2 $ft-lb$
(e) 120 $ft-lb$

Exercise

A spring has a natural length of 2 m. If a force of 12 N is required to hold the spring to a length of 4 m, find the work done to stretch the spring from 3 m to 5 m.

(a) 24 J

(b) 27 J

(c) 30 J

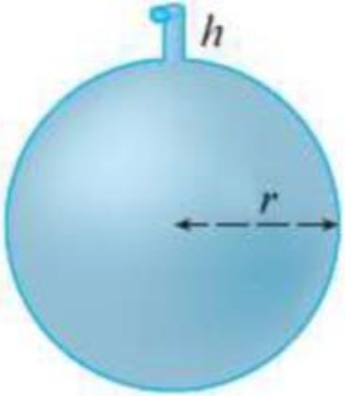
(d) 12 J

(e) 6 J

A spherical tank with radius 5 m is half full of a liquid that has a density of 1000 kg/m^3. The tank has a 1 m spout at the top. Set up an integral to find the work required to pump all the water out of the spout. (Use 9.8 m/s^2 for g.)

Note 1. Do NOT evaluate your integral.

Note 2. Clearly indicate in the picture below where you are placing your axis and which direction is positive.



Exercise

A 10 foot rope weighing 50 pounds hangs vertically from a tall building. There is a 15 pound weight attached to the end of the rope. How much work is done in lifting the rope and the weight to the top of the building?

(a) 400 foot pounds

(b) 7650 foot pounds

(c) 235 foot pounds

(d) 250 foot pounds

(e) None of these

A force of 30 N is required to stretch a spring from its natural length of 1 m to a length of 3 m. How much work is done in stretching the spring from 2 m to 4 m?

(a) 60 Joules

(b) $\frac{221}{2}$ Joules

(c) 240 Joules

(d) 360 Joules

(e) 90 Joules

A cylindrical shaped tank is filled with water to a depth of 8 m. The tank has a height of 20 meters and a radius of 4 meters. Set up but **do not evaluate** an integral that gives the work required to pump all of the water out of a 1 meter tall spout top located at the of the tank. Use $\rho g = 9800 N/m^3$ for the weight density of water. **Clearly indicate where you are placing the axis and which direction is positive.**

Exercise

A force of 30 N is required to stretch a spring from its natural length of 1 m to a length of 3 m. How much work is done in stretching the spring from 2 m to 4 m?

(a) 60 Joules

(b) $\frac{221}{2}$ Joules

(c) 240 Joules

(d) 360 Joules

(e) 90 Joules

Exercise

Compute $\int x^3 \sin(x)\ dx$

(a) $C - x^3\cos(x) - 3x^2\sin(x) + 6x\cos(x) + 6\sin(x)$

(b) $C - x^3\cos(x) + 3x^2\sin(x) + 6x\cos(x) - 6\sin(x)$

(c) $C + x^3\cos(x) + 3x^2\sin(x) - 6x\cos(x) + 6\sin(x)$

(d) $C + x^3\cos(x) + 3x^2\cos(x) - 6x\sin(x) + 6\cos(x)$

(e) $C + \frac{1}{4}x^4 - \frac{1}{3}x^3\sin(x) + \frac{1}{6}x^2\cos(x) + \frac{1}{6}x\sin(x)$

Compute $\int\limits_1^e x^2 \ln x\ dx.$

(a) $\frac{2}{9}e^3 - \frac{1}{9}$

(b) $\frac{2}{9}e^3 + \frac{1}{9}$

(c) $1 - e$

(d) $e^2 - \frac{1}{9}e^3 + \frac{1}{9}$

(e) None of these.

Evaluate $\int_1^2 \ln x\, dx$

(a) $2\ln 2 - 1$

(b) $\frac{\ln(2)}{2} - 1$

(c) $\frac{\ln(2)}{2} - \frac{3}{2}$

(d) $2\ln 2 - 3$

(e) $-\frac{1}{2}$

Compute $\int_0^1 xe^{-x}\, dx$

(a) $1 + 2e^{-1}$

(b) $\frac{1}{2} - \frac{1}{2}e^{-1}$

(c) $2e^{-1} - 1$

(d) $-\frac{1}{2} + \frac{1}{2}e^{-1}$

(e) $1 - 2e^{-1}$

Compute $\int_0^1 (x^2+3)e^{-x}\ dx.$

(a) $-\frac{8}{e}$

(b) $\frac{2}{e} - 1$

(c) $\frac{2}{e} + 1$

(d) $-\frac{2}{e} + 5$

(e) $-\frac{8}{e} + 5$

Compute $\int_1^e (\ln x)^2\ dx.$

(a) 1

(b) $\frac{1}{e} - 1$

(c) $\frac{2}{e} - 2$

(d) $e - 1$

(e) $e - 2$

Exercise

Find $\int e^{2x} \cos x \, dx$

$$\int_0^1 (x+2)\cos x \, dx =$$

(a) $3\sin(1) + \cos(1) - 1$

(b) $3\sin(1) + \cos(1)$

(c) $3\sin(1) - \cos(1) - 1$

(d) $3\sin(1) - \cos(1)$

(e) None of these