$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = (-1).x^{-1-1} = (-1).x^{-2} = \frac{-1}{x^2}$

x^n

TEXAS A&M UNIVERSITY
College of Arts & Sciences

MATHEMATICS
Math 142 - Spring 2024
Week in Review 4: 2.3, 2.4

$\ln(e) = 1$

$\frac{d}{dx}(f(x)) = f'(x)$ $\quad$ $\frac{d}{dx}(c) = 0$

Problem 1. Find the derivative $\left(ie\ \frac{df}{dx}\right)$ for the following:

① $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$ $\quad$ $\frac{d}{dx}(x) = 1$

② $\frac{d}{dx}(a^x) = a^x . \ln a$ $\quad$ $\frac{d}{dx}(e^x) = e^x$

③ $\frac{d}{dx}(\log_b(x)) = \frac{1}{x . \ln b}$ $\quad$ $\frac{d}{dx}\ln(x) = \frac{1}{x}$

(1) $f(x) = 2x^5 + 3\sqrt[3]{x^{10}} + 6e^x - 7^x - 5\ln(x) + \underbrace{\log_3(x)} + 4\pi x + \underline{\underline{3e}}$

$2(x^5)$ $\quad$ $3(x^{10/3})$

$f'(x) = 2(5x^4) + \cancel{3}\left(\frac{10}{\cancel{3}}x^{\left(\frac{10}{3}-1\right)}\right) + \underbrace{6(e^x . \ln \cancel{e}^{1})} - 7^x . \ln 7 - 5\left(\frac{1}{x \ln \cancel{e}^{1}}\right) + \frac{1}{x\ln 3} + 4\pi(1) + 0$

$f'(x) = 10x^4 + 10x^{7/3} + 6e^x - 7^x . \ln 7 - \frac{5}{x} + \frac{1}{x\ln 3} + 4\pi$

(2) $f(x) = \left(\frac{7}{x^2} - 4x^3\right)(3x^2 - 5x)$ $= \frac{7}{\cancel{x^2}} . 3\cancel{x^2} + \frac{7}{\cancel{x^2}_{x}}(-5\cancel{x}) + (-4x^3)(3x^2) + (-4x^3)(-5x)$

$x^3 . x^2 = x^5$

$\frac{1}{x} = x^{-1}$

$\frac{d}{dx}(x^{-1}) = (-1)x^{-2} = -\frac{1}{x^2}$

$f(x) = 21 - \frac{35}{x} - 12x^5 + 20x^4$ $\quad$ $-35 . x^{-1}$

$f'(x) = 0 - 35(-1)x^{-2} - 12(5x^4) + 20(4x^3)$

$f'(x) = \frac{35}{x^2} - 60x^4 + 80x^3$

(3) $f(x) = \frac{x^3 + 5x^2 + 7x + \sqrt[5]{x^2}}{3x}$

$= \frac{x^3}{3x} + \frac{5x^2}{3x} + \frac{7x}{3x} + \frac{x^{2/5}}{3x}$

$= \frac{1}{3}x^2 + \frac{5}{3}.x + \underbrace{\frac{7}{3}} + \frac{1}{3}.x^{\frac{2}{5}-1}$ $\quad$ $\frac{1}{3}x^{-3/5}$

$f'(x) = \frac{1}{3}(2x^1) + \frac{5}{3}(1) + 0 + \frac{1}{\cancel{3}}\left(-\frac{\cancel{3}}{5}\right)x^{-\frac{3}{5}-1}$

$= \frac{2x}{3} + \frac{5}{3} - \frac{1}{5}x^{-8/5}$

$\log(10) = 1.$ $\qquad$ $\ln_a(a) = 1.$

$\ln e = 1.$ $\qquad$ $\log(x) \to \log_{10}(x)$

$\log_2(2) = 1.$ $\qquad$ $\ln(x) \to \log_e(x)$

$$\frac{d}{dx}(h\cdot g) = h\cdot g' + gh' = gh' + hg'$$

Product Rule

2

Problem 2. Find the derivative of $f(x) = \underbrace{(2x^2 + 4x + 11)}_{h}\underbrace{(2^x + 7 - 3x^7)}_{g}$. Do not simplify your answer.

$$f'(x) = (2x^2+4x+11)(2^x\cdot\ln 2 + 0 - 3(7x^6)) + (2^x+7-3x^7)(2\cdot(2x) + 4(1) + 0)$$

$$= (2x^2+4x+11)(2^x\ln 2 - 21x^6) + (2^x+7-3x^7)(4x+4)$$

$$\frac{d}{dx}\left(\frac{hi}{lo}\right) = \frac{lo\,d(hi) - hi\,d(lo)}{lo\cdot lo}$$

quotient rule.

Problem 3. Find the derivative of $f(x) = \dfrac{10x^5 - 3x^2 + 4e^x}{4x^4 - \sqrt{x^3} - 2\ln x}$. Do not simplify your answer.

$$f'(x) = \frac{(4x^4-\sqrt{x^3}-2\ln x)(50x^4 - 6x + 4e^x) - (10x^5-3x^2+4e^x)(16x^3 - \frac{3}{2}\sqrt{x} - 2\frac{1}{x})}{(4x^4-\sqrt{x^3}-2\ln x)^2}$$

$\sqrt{x} = x^{1/2}$

$\sqrt[2]{x^3} = x^{3/2}$

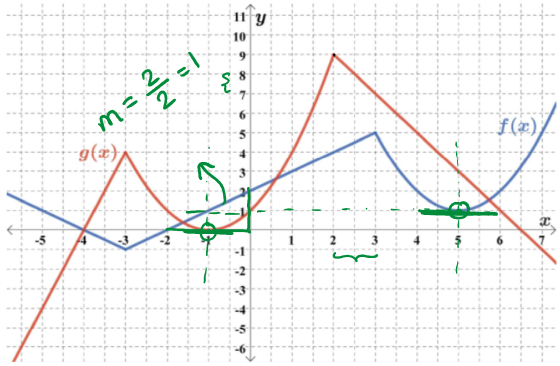
$\frac{d}{dx}(x^{3/2}) = \frac{3}{2}x^{\frac{3}{2}-1}$

$= \frac{3}{2}x^{1/2}$

$= \frac{3}{2}\sqrt{x}$

Problem 4. The graphs of f and g are given below. Use the graphs to answer the following:

$f(-1)=1$
$g(-1)=0$
$f'(-1)=1$
$g'(-1)=0$



horizontal tangent, $f'(x)=0$.

$f(5)=1$

$f'(5)$ = slope of tangent line at $x=5$

$f'(5)=0$

(1) If $h(x) = \underbrace{3e^x}\underbrace{(f(x)-4x^2)}$, find $h'(5)$.

$$h'(x) = 3e^x(f(x)-4x^2) + 3e^x(f'(x)-8x)$$

$$h'(5) = 3e^5(f(5) - 4\cdot5^2) + 3e^5(f'(5) - 8\cdot5)$$

$$= 3e^5(1-100) + 3e^5(0-40)$$

$$= 3e^5(-99) + 3e^5(-40)$$

$$= 3e^5(-99-40) = 3e^5(-139) = -417e^5$$

(2) If $k(x) = \dfrac{x^2 - \cancel{\sqrt{x}}\, x^5}{2f(x)-g(x)}$ find $k'(-1)$.

$$k'(x) = \frac{[2f(x)-g(x)](2x-5x^4) - (x^2-x^5)[2f'(x)-g'(x)]}{[2f(x)-g(x)]^2}$$

$$k'(-1) = \frac{[2\overset{1}{f(-1)} - \overset{0}{g(-1)}](2\cdot(-1) - 5(-1)^4) - ((-1)^2 - (-1)^5)[2\overset{1}{f'(-1)} - \overset{0}{g'(-1)}]}{[2\overset{1}{f(-1)} - \overset{0}{g(-1)}]^2}$$

$$= \frac{[2(1)-0](-2-5) - \underbrace{(1-(-1))}_{1+1=2}[2(1)-0]}{[2(1)-0]^2} = \frac{2(-7)-(2)(2)}{2^2}$$

$$k'(-1) = \frac{-14-4}{4} = -\frac{18}{4} \text{ Ans.}$$

$$k'(-1) = \frac{-14-4}{4} = -\frac{18}{4} \text{ Ans.}$$

4

Problem 5. Given that $f(3) = 2,\ g(3) = -5,\ f'(3) = 7,\ g'(3) = 9$, find $h'(3)$ for the following:

(1) $h(x) = 7f(x) - 3g(x)$

$$h'(x) = 7f'(x) - 3g'(x)$$

$$h'(3) = 7\underbrace{f'(3)}_{7} - 3\underbrace{g'(3)}_{9} = 7(7) - 3(9)$$

$$= 49 - 27 = 22$$

(2) $h(x) = 2f(x)g(x)$

$$h'(x) = 2f(x)g'(x) + 2f'(x)g(x)$$

$$h'(3) = 2\underbrace{f(3)}_{2}\,\underbrace{g'(3)}_{9} + 2\underbrace{f'(3)}_{7}\,\underbrace{g(3)}_{-5} = 2(2)(9) + 2(7)(-5)$$

$$= 36 - 70$$

$$= -34$$

(3) $h(x) = \dfrac{f(x)}{4 + g(x)}$

$$h'(x) = \frac{(4+g(x))f'(x) - f(x)(0 + g'(x))}{(4+g(x))^2}$$

$$h'(3) = \frac{(4+\overset{-5}{g(3)})\overset{7}{f'(3)} - \overset{2}{f(3)}\,\overset{9}{g'(3)}}{[4+\underset{-5}{g(3)}]^2} = \frac{(4-5)7 - 2(9)}{(4-5)^2}$$

$$= \frac{-7-18}{(-1)^2} = \frac{-25}{1}$$

$$= -25$$

(4) $h(x) = \dfrac{3x^2g(x)}{f(x)}$

$(3x^2)(g(x)) \rightarrow$ product rule

$$h'(x) = \frac{f(x)[3(2x)g(x) + 3x^2 \cdot g'(x)] - 3x^2g(x)\cdot f'(x)}{[f(x)]^2}$$

$$h'(3) = \frac{\overset{2}{f(3)}[6\cdot 3\cdot \overset{(-5)}{g(3)} + 3\cdot 9\cdot \overset{9}{g'(3)}] - 3\cdot 9\cdot \overset{(-5)}{g(3)}\cdot \overset{7}{f'(3)}}{[\underset{2}{f(3)}]^2}$$

$$= \frac{2(-90 + 243) + 945}{4} = \frac{1251}{4}$$

$f(0) = \frac{0+0-4}{0-3} = \frac{-4}{-3} = \frac{4}{3}$

Problem 6. Given $f(x) = \dfrac{x^2+x-4}{x-3}$,

(1) For what value(s) of x does the graph of $f(x)$ have a horizontal tangent line?

$f'(x) = 0$

$$f'(x) = \frac{(x-3)(2x+1) - (x^2+x-4)(1)}{(x-3)^2} = 0$$

set only numerator to 0 & solve for x

$$2x^2 + \cancel{x} - 6x - 3 - x^2 - \cancel{x} + 4 = 0$$

$$x^2 - 6x + 1 = 0$$

quad formula.

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{+6 \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)} = \frac{6 \pm \sqrt{36-4}}{2}$$

$$x = \frac{6 \pm \sqrt{32}}{2}$$

Horizontal tangent lines at $x = \frac{6+\sqrt{32}}{2}$ and $x = \frac{6-\sqrt{32}}{2}$

(2) Find the equation of the tangent line to $f(x)$ at $x = 0$.

pt & slope

$y - y_1 = m(x - x_1)$

$x = 0$

$f(0) = \frac{4}{3}$ $\Big\}$ $\boxed{(0, \frac{4}{3})}$

$\boxed{m = f'(0) = \frac{1}{9}}$

$f'(x) = \frac{x^2 - 6x + 1}{(x-3)^2}$

$f'(0) = \frac{0 - 0 + 1}{(0-3)^2} = \frac{1}{9}$

$y - \frac{4}{3} = \frac{1}{9}(x - 0)$

$$\boxed{y = \frac{1}{9}x + \frac{4}{3}}$$

$$(876)\ (0.9985)^x$$
$\downarrow$
exp. f^n

Problem 7. The monthly price demand equation for a set of high quality knives is given by $p = 876(0.9985)^x$ dollars, where x is the number of knife sets bought each month.

(1) Find the marginal revenue function.

need $R'(x)$ $\longrightarrow$ $R(x) = px = \underbrace{876x}\cdot\underbrace{(0.9985)^x}$

product rule

$$R'(x) = (876)(0.9985)^x + (876x)(0.9985)^x \cdot \ln(0.9985)$$

(2) Find the marginal revenue at a production level of 800 kinfe sets. Interpret your answer.

$x = 800$

$R'(800) = -52.96$ \$ / set

At a production level of 800 knife sets, revenue is decreasing at a rate of \$52.96 /set.

(3) Approximate the revenue from selling the 500^{th} knife set.

single item

$$R'(499) = \begin{cases} ① \ Y_1 = 876x\,(0.9985)^x \\ ② \text{ Math } \# 8 \longrightarrow \dfrac{d}{d\boxed{x}}\boxed{Y_1}\Big|_{x=\boxed{499}} \end{cases} = \$103.93$$

(4) Find the exact revenue from selling the 500^{th} knife set.

$$R(500) - R(499)$$
$$Y_1(500) - Y_1(499)$$
$$= \$\,103.55$$

$f(x) = e^x - x^2$

Problem 8. Find the equation of the tangent line to the curve $y = e^x - x^2$ at $x = 1$.

pt

slope

$m = f'(1)$

$x=1$

$f(1) = e^1 - 1^2$
$= e - 1$

$(1, e-1)$

$f'(x) = e^x - 2x$

$f'(1) = e^1 - 2(1)$

$m = e - 2$

$y - y_1 = m(x - x_1)$

$y - (e-1) = (e-2)(x-1)$

$= (e-2)x - (e-2)$

$y = (e-2)x - (e-2) + (e-1)$

$y = (e-2)x + 1$

Problem 9. Find the value(s) of x where the tangent line to the graph of $f(x) = 2x^5 - 30x^3 + e^2$ is horizontal.

$f'(x) = 0$

$f(x) = 2x^5 - 30x^3 + e^2$

$f'(x) = 2(5x^4) - 30(3x^2) + 0$

$= 10x^4 - 90x^2$

$= 10x^2(x^2 - 9)$

$f'(x) = 10x^2(x+3)(x-3) = 0$

$x = 0$ $\quad$ $x+3=0$ $\quad$ $x-3=0$

$x = -3$ $\quad$ $x = 3$

Horizontal tangent lines at $x = -3$, $x = 0$ and $x = 3$