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TEXAS A&M UNIVERSITY
Mathematics

Math 140 - Spring 2024
Week In Review ♯9 - Apr. 01, 2024

Section 5.3: ~~Power and Radical~~ Rational Functions

- Domain - denominator can not be zero
- Rationalizing ★
- Difference Quotient

$\frac{p(x)}{q(x)}$, require $q(x) \neq 0$.

$\frac{f(x+h)-f(x)}{h}$

$x^2 - a^2 = (x+a)(x-a)$

Pr 1. Compute each of the following:.

(a) $\left(\frac{(x+2)^2}{x^2-16}\right)\left(\frac{7(x+4)}{(x-4)}\right) = \frac{7(x+2)^2(x+4)}{(x+4)(x-4)(x-4)}$

"simplify"

$= \boxed{\frac{7(x+2)^2}{(x-4)^2}}$

graph $x = \frac{3}{2}$ $(x - \frac{3}{2})$

$4x^2 - 12x + 9 = (2x-3)(2x-3) = (2x-3)^2$

(b) $\left(\frac{4x^3 - 12x^2 + 9x}{x^2-49}\right)\left(\frac{10x^2-15x}{x^2+4x-21}\right) = \frac{x(4x^2-12x+9) \cdot 5x(2x-3)}{(x-7)(x+7)(x+7)(x-3)} = \boxed{\frac{5x^2(2x-3)^3}{(x-3)(x-7)(x+7)^2}}$

$\frac{3}{3}\cdot\frac{7}{10} + \frac{4}{15}\cdot\frac{2}{2} =$

$10 = 2 \cdot 5$ $15 = 3 \cdot 5$

(c) $\frac{2}{(x-2)(x+4)} - \frac{x+5}{(x-2)(x+2)}$

$= \frac{2(x+2)}{(x-2)(x+4)(x+2)} - \frac{(x+5)(x+4)}{(x-2)(x+2)(x+4)}$

$= \frac{2x+4-(x^2+4x+5x+20)}{(x-2)(x+2)(x+4)} = \frac{2x+4-x^2-4x-5x-20}{(x-2)(x+2)(x+4)} = \boxed{\frac{-x^2-7x-16}{(x-2)(x+2)(x+4)}}$

numerator $= 0$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-1)(-16)}}{2(-1)}$

$= \frac{7 \pm \sqrt{49-64}}{-2}$ → negative

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Pr 2. State the domain of each rational function. Then classify each each domain restriction as the location of a hole or vertical ~~asymptote~~ on the graph of the function.

domain: denominator not zero

$x = c$ is a "hole" if $(x-c)$ is in the numerator + denominator (the same number of times) otherwise, asymptote.

(a) $f(x) = \frac{(3x-2)(2x-5)}{(x-5)(2x+5)}$

Denominator is $(x-5)(2x+5)$

$(x-5)(2x+5) = 0$ → $x-5=0$ or $2x+5=0$

$x = 5$ or $2x = -5$, $x = -5/2$

$-5/2$ 0 5

Domain: $(-\infty, -5/2) \cup (-5/2, 5) \cup (5, \infty)$

vertical asymptotes at $x = -5/2$, $x = 5$

(b) $g(x) = \frac{(x+3)(x-2)}{(x-2)(x+2)}$

$(x-2)(x+2) = 0$

$x-2=0$ $x+2=0$

vertical asymptotes at x = -1/2, x = 3 otherwise,

asymptote.

(b) $g(x) = \frac{(x+3)(x-2)}{(x-2)(x+2)}$

Do not cancel for Domain questions

$(x-2)(x+2) = 0$

$x-2=0 \qquad x+2=0$

$x=2 \qquad x=-2$

Domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

$x=2$ is a hole, $x=-2$ is a vertical asymptote

(c) $h(x) = \frac{-2x}{6x^2-8x} = \frac{-2x}{2x(3x-4)}$

Domain: $2x(3x-4) \neq 0$

$2x=0$ or $3x-4=0$

$x=0 \qquad 3x=4$

$x=4/3$

$(-\infty,0)\cup(0,4/3)\cup(4/3,\infty)$

$x=0$ is a hole, $x=4/3$ is a vertical asymptote.

(d) $j(x) = \frac{3x^2-6x+3}{x^2-9} = \frac{3(x^2-2x+1)}{(x+3)(x-3)} = \frac{3(x-1)^2}{(x+3)(x-3)}$

Domain: $(x+3)(x-3) \neq 0$

$x+3=0$ or $x-3=0$

$x=-3$ or $x=3$

$(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$x=-3$, $x=3$ are vertical asymptotes.

y-intercept $(0, f(0))$

x-intercept $(x, 0)$ → numerator is 0, denominator is not 0.

Pr 3. Determine the x- and y-intercepts, if possible, for each function.

(a) $f(x) = \dfrac{(3x-2)(2x-5)}{(x-5)(2x+5)}$ → solve $(3x-2)(2x-5)=0$

$3x-2=0$ or $2x-5=0$

$3x=2$ $\quad 2x=5$

$x=2/3$ $\quad x=5/2$

$\boxed{\text{x-intercepts } (2/3, 0) \text{ and } (5/2, 0)}$

y-intercept: $x=0$ $\quad (0, -2/5)$

$\dfrac{(3\cdot0-2)(2\cdot0-5)}{(0-5)(2\cdot0+5)} = \dfrac{(-2)(-5)}{(-5)(5)} = -2/5$

(b) $g(x) = \dfrac{(x+3)(x-2)}{(x-2)(x+2)}$

$\boxed{\text{x-intercepts : } (-3, 0)}$

$(x+3)(x-2)=0$

$x+3=0$ or $x-2=0$

$x=-3$ or $\cancel{x=2}$ not in domain

y-intercept: $x=0$ (erase x)

$\dfrac{(+3)(-2)}{(-2)(+2)} = \dfrac{3}{2}$

$(0, 3/2)$

(c) $h(x) = \dfrac{-2x}{6x^2-8x} = \dfrac{-2x}{2x(3x-4)}$

x-intercept: No x-intercept

$-2x=0$

$x=0$ not in the domain

y-intercept: $x=0$ (not in the domain) No y-intercept

(d) $j(x) = \dfrac{3x^2-6x+3}{x^2-9} = \dfrac{3(x-1)^2}{(x-3)(x+3)}$

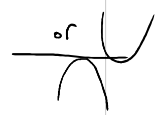
$\boxed{\text{x-intercepts: } (1,0)}$

$3(x-1)^2=0$

$(x-1)=0$

$x=1$

or



y-intercept: $x=0$ $\quad (0, -1/3)$

$\dfrac{3\cdot(0-1)^2}{(0-3)(0+3)} = \dfrac{\cancel{3}\cdot(-1)^2}{(-3)(\cancel{3})} = \dfrac{1}{-3} = -\dfrac{1}{3}$

$\dfrac{\cancel{3}}{\cancel{3}} = 1 \ne 0$

remove h from denominator

$$\frac{f(x+h)-(f(x))}{h}$$

$$\frac{2+1}{2} \neq \frac{1}{1}$$

Pr 4. Compute and simplify the difference quotient for each function.

(a) $f(x) = -x^2 + 5x - 4$

$(x+a)^2 = x^2 + 2ax + a^2$

$(x+h)^2 \neq x^2 + h^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{-(x+h)^2 + 5(x+h) - 4 - (-x^2 + 5x - 4)}{h}$$

$$= \frac{-(x^2 + 2hx + h^2) + 5x + 5h - 4 + x^2 - 5x + 4}{h}$$

$$= \frac{-x^2 - 2hx - h^2 + 5x + 5h - 4 + x^2 - 5x + 4}{h}$$

$$= \frac{-2hx - h^2 + 5h}{h} = \frac{h(-2x - h + 5)}{h}$$

$$= \boxed{-2x - h + 5}$$

(b) $g(x) = \frac{3x}{2x - 2}$

$$\frac{g(x+h) - g(x)}{h} = \frac{\frac{3(x+h)}{2(x+h)-2} - \left(\frac{3x}{2x-2}\right)}{h}$$

→ main step: common denominator

$$= \frac{\frac{3x+3h}{2x+2h-2} - \frac{3x}{2x-2}}{h} = \frac{\frac{(3x+3h)(2x-2)}{(2x+2h-2)(2x-2)} - \frac{3x(2x+2h-2)}{(2x-2)(2x+2h-2)}}{h}$$

$$\frac{f(x)}{h} = \frac{1}{h} \cdot [f(x)]$$

$$= \frac{1}{h}\left[\frac{(3x)(2x) + 3x(-2) + (3h)(2x) + (3h)(-2) - (3x(2x) + 3x(2h) + 3x(-2))}{(2x+2h-2)(2x-2)}\right]$$

$$= \frac{1}{h} \cdot \frac{\cancel{6x^2} - \cancel{6x} + \cancel{6xh} - 6h - \cancel{6x^2} - \cancel{6xh} + \cancel{6x}}{(2x+2h-2)(2x-2)}$$

$$= \frac{1}{\cancel{h}} \cdot \frac{-6\cancel{h}}{(2x+2h-2)(2x-2)} = \frac{-6}{(2x+2h-2)(2x-2)}$$

$f(x) = x^r$

$x^{1/n}$

$f(x)^{a/b} = \sqrt[b]{f(x)^a}$

Section 5.4: Power and Radical Functions

- Power Functions
- Radical Functions
- Domain of Radical Functions based on Index
- Conjugate
- Rationalizing a numerator or denominator

Pr 1. Rewrite each radical in its equivalent exponent (power) form, assuming x is in the domain of each function.

5 (a) $\sqrt[5]{-2x^2+4x}$ $= (-2x^2+4x)^{1/5}$

(b) $6\sqrt[3]{3x^2-8x+2}$ $= 6(3x^2-8x+2)^{1/2}$

(c) $\sqrt[4]{(2-5x)^3}$ $(2-5x)^{\frac{3}{4}}$

Pr 2. Rewrite each exponent function in its equivalent radical form, assuming x is in the domain of each function.

(a) $(x^2+3x)^{7/11}$ $= \sqrt[11]{(x^2+3x)^7}$ $\sqrt[n]{x} = y$ if $y^n = x$.

(b) $(3x+8)^{9/13}$ $= \sqrt[13]{(3x+8)^9}$

(c) $2(5x-3)^{7/3}$ $= 2\sqrt[3]{(5x-3)^7}$

Pr 3. State the domain of each function. Write your answer using interval notation. Then determine the x- and y-intercepts, if possible.

(a) $f(x) = \sqrt[6]{3x - 28}$

6 is even

domain: $\sqrt[2k]{f(x)}$

require $f(x) \geq 0$

$\sqrt[2k+1]{f(x)}$ no restrictions

require $3x - 28 \geq 0$

$3x \geq 28$

$x \geq \frac{28}{3}$

Domain: $\left[\frac{28}{3}, \infty\right)$

x-intercept: $3x - 28 = 0$

$3x = 28$

$x = \frac{28}{3}$

$(28/3, 0)$ ✓

y-intercept: Does not exist

0 is not in the domain

(b) $g(x) = 2\sqrt[5]{x - 5}$

5 is odd

domain of $g(x)$ = domain of $x-5$

$= (-\infty, \infty)$

x-intercept: $(5, 0)$

$x - 5 = 0 \quad x = 5$

y-intercept: $x = 0$

$g(0) = 2\sqrt[5]{0-5} = 2\sqrt[5]{-5}$

$(0, 2\sqrt[5]{-5})$ or $(0, 2\cdot(-5)^{1/5})$

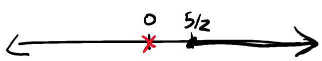
(c) $h(x) = 5(2x - 5)^{5/12}$

5 is odd, 12 is even

require $2x - 5 \geq 0$

$2x \geq 5$

$x \geq 5/2$



Domain: $[5/2, \infty)$

what if $h(x) = (-2x+3)^{1/2}$

$-2x + 3 \geq 0$

$\frac{-2x}{-2} \geq \frac{-3}{-2}$

$x \leq \frac{3}{2}$

$(-\infty, 3/2]$

not

$[3/2, \infty)$

x-intercept: $2x - 5 = 0 \rightarrow 2x = 5, \quad x = 5/2$

$(5/2, 0)$

y-intercept: $h(0) =$ DNE

no y-intercept.

Pr 4. State the domain of each function. Write your answer using interval notation.

(a) $f(x) = (3x-4)^{-4/3}$ $=$ $\sqrt[3]{(3x-4)^{-4}} = \sqrt[3]{\dfrac{1}{(3x-4)^4}}$

3 is odd

Domain $f(x)$ = Domain $\dfrac{1}{(3x-4)^4}$

need $3x-4 \neq 0$

$3x \neq 4$

$x \neq 4/3$

$\boxed{(-\infty, 4/3) \cup (4/3, \infty)}$

Similar example: $(3x-4)^{-1/4} \rightarrow \sqrt[4]{\dfrac{1}{3x-4}} \rightarrow > 0$ $\boxed{(4/3, \infty)}$

(b) $g(x) = \dfrac{\sqrt{x+2}}{5\sqrt[3]{x+2}}$ $= \dfrac{1}{5} \dfrac{(x+2)^{1/2}}{(x+2)^{1/3}} = \dfrac{1}{5}(x+2)^{1/2}(x+2)^{-1/3}$

$\dfrac{1}{a^b} = a^{-b}$

assumes $a \neq 0$

$= \dfrac{1}{5}(x+2)^{1/2 - 1/3} = 1/5\,(x+2)^{1/6}$

Denominator is 0 when $x=-2$

even root $x+2 \geq 0$ $x \geq -2$

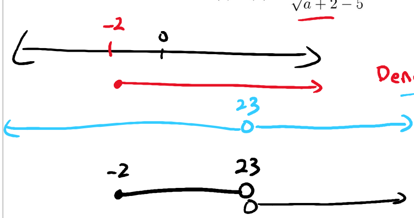
$[-2, \infty)$

$\boxed{\text{correct Domain: } (-2, \infty)}$

(c) $h(a) = \dfrac{3a}{\sqrt{a+2} - 5}$

$\sqrt{a+2} \rightarrow$ forces $a+2 \geq 0 \rightarrow a \geq -2$

$[-2, \infty)$



Denominator $= 0$ $\sqrt{a+2} - 5 = 0$

$\sqrt{a+2} = 5$

$a+2 = 5^2$

$a = 5^2 - 2 = 25 - 2 = 23$

$a \neq 23$

$\boxed{\text{Domain: } [-2, 23) \cup (23, \infty)}$

$$\frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b} \ldots$$

Pr 5. Rationalize each numerator or denominator, as appropriate, and simplify the expression.

(a) $\frac{3x}{\sqrt{x}-3}$

" multiply by the conjugate"

$a+b\sqrt{D} \longrightarrow a-b\sqrt{D}$

$(a^2-Db^2) = (a-b\sqrt{D})(a+b\sqrt{D})$

$$\frac{3x}{(\sqrt{x}-3)}\frac{(-\sqrt{x}-3)}{(-\sqrt{x}-3)} = \frac{3x(-\sqrt{x}-3)}{-x\cancel{-3\sqrt{x}}\cancel{+3\sqrt{x}}+9}$$

$$= \boxed{\frac{3x(-\sqrt{x}-3)}{-x+9}} = \frac{3x(\sqrt{x}+3)}{x-9}$$

"rationalize the numerator"

(b) $\frac{\sqrt{3x-2}+5}{1}$

$$\frac{(\sqrt{3x-2}+5)}{1}\frac{(-\sqrt{3x+2}+5)}{(-\sqrt{3x-2}+5)} = \frac{-(3x+2)+5^2}{-\sqrt{3x-2}+5}$$

$$= \frac{-3x-2+25}{-\sqrt{3x-2}+5}$$

$$\boxed{= \frac{-3x+23}{-\sqrt{3x-2}+5}} = \frac{3x-23}{\sqrt{3x-2}-5}$$

(c) $\frac{(\sqrt{x+h}-\sqrt{x})}{h}\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})} = \frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$

$$= \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$

Pr 6. Compute and simplify the difference quotient for $F(x) = 3\sqrt{2-x}$. → key step multiply numerator/den. by conjugate

$$\frac{F(x+h)-F(x)}{h} = \frac{1}{h}\left[3\sqrt{2-(x+h)} - \left(3\sqrt{2-x}\right)\right]$$

$$= \frac{3}{h}\left[\sqrt{2-x-h} - \sqrt{2-x}\right]$$

$$= \frac{3}{h}\left[\sqrt{2-x-h} - \sqrt{2-x}\right]\frac{\left(\sqrt{2-x-h} + \sqrt{2-x}\right)}{\left(\sqrt{2-x-h} + \sqrt{2-x}\right)}$$

$$= \frac{3}{h}\left(\frac{2-x-h-(2-x)}{\sqrt{2-x-h} + \sqrt{2-x}}\right)$$

$$= \frac{3}{\cancel{h}}\left(\frac{-\cancel{h}}{\sqrt{2-x-h} + \sqrt{2-x}}\right) = \boxed{\frac{-3}{\sqrt{2-x-h} + \sqrt{2-x}}}$$