Session 4: Sections 2-3 and 2-4

Introductory Derivative Rules

Constant:	$\frac{d}{dx}(k) = 0$	where k is any real number
Power:	$\frac{d}{dx}(x^n) = nx^{n-1}$	where n is any real number
Special Case:	$\frac{d}{dx}(x) = 1$	(because $x = x^1$)
Exponential:	$\frac{d}{dx}(b^x) = b^x \ln(b)$	where b is any positive real number
Special Case:	$\frac{d}{dx}(e^x) = e^x$	(because $\ln(e) = 1$)
Logarithm:	$\frac{d}{dx}(\log_b(x)) = \frac{1}{x\ln(b)}$	where b is any positive real number
Special Case:	$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$	(because $\ln(e) = 1$)
Sum/Difference:	$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$	where f and g are differentiable functions
Constant Multiple:	$\frac{d}{dx}(k \cdot f(x)) = k\left(\frac{d}{dx}(f(x))\right)$	where k is any real number and f is a differentiable function

1. Find $f'(t)$ given $f(t) = 2t^2 - 3t + 1$.

2. Given $y = \frac{5}{9t^6} + 6\sqrt[3]{t^2}$, find $\frac{dy}{dt}$.

3. Find $\dfrac{d}{dx}\left(\pi x^{2\pi} + \dfrac{5x^8}{\sqrt{x}} + \dfrac{3e}{\sqrt[6]{x^5}}\right)$.

4. Given $f(x) = 3e^x + 4\ln(x) - \dfrac{1}{2}\log_7(x)$, find $f'(x)$.

5. Find $\dfrac{dp}{dx}$ given $p = 10^x + x^7 + \log(x^5) + 10e^x$.

6. Find $f'(x)$ given $f(x) = \ln\left(\frac{x^2}{5}\right) + \log(2x)$.

7. If $h(x) = -4f(x) + 5g(x) - 9$, $f'(5) = 8$, and $g'(5) = 4$, find $h'(5)$.

8. Given $f(x)$ is a polynomial function such that $f(2) = 17$ and $f'(x) = 12x^3 - 12x$, find the equation of the tangent line at $x = 2$.

Estimating the Cost/Revenue/Profit of a Single Item Using Marginal Analysis

- The approximate cost of making the n^{th} item is $C\,'(n-1)$.
- The approximate revenue from selling the n^{th} item is $R\,'(n-1)$.
- The approximate profit from making and selling the n^{th} item is $P\,'(n-1)$.

The Exact Cost/Revenue/Profit of a Single Item

- The exact cost of making the n^{th} item is $C(n) - C(n-1)$.
- The exact revenue from selling the n^{th} item is $R(n) - R(n-1)$.
- The exact profit from making and selling the n^{th} item is $P(n) - P(n-1)$.

9. The total profit (in dollars) from the sale of x skateboards is $P(x) = 30x - 0.3x^2 - 250$ for $0 \leq x \leq 100$.

   (a) Find the exact profit from the sale of the 26^{th} skateboard.

   (b) Find the marginal profit function and then use it to approximate the profit from the sale of the 26^{th} skateboard.

For the rules below, assume f and g are differentiable functions.

The Product Rule:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\left(\frac{d}{dx}\left(g(x)\right)\right) + g(x)\left(\frac{d}{dx}\left(f(x)\right)\right)$$

$$= f(x)\cdot g\,'(x) + g(x)\cdot f\,'(x)$$

The Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\left(\frac{d}{dx}\left(f(x)\right)\right) - f(x)\left(\frac{d}{dx}\left(g(x)\right)\right)}{\left(g(x)\right)^2}$$

$$= \frac{g(x)\cdot f\,'(x) - f(x)\cdot g\,'(x)}{\left(g(x)\right)^2}$$

10. Find $f'(x)$ for the following functions. You do not need to simplify the functions after applying the derivative rules.

    (a) $f(x) = x^2\ln(x)$

    (b) $f(x) = \dfrac{4e^x}{7x^3 + 2x^2 + 5x}$

(c) $f(x) = \dfrac{\log_3(x^6)}{e^2 + \sqrt[5]{x^3}}$

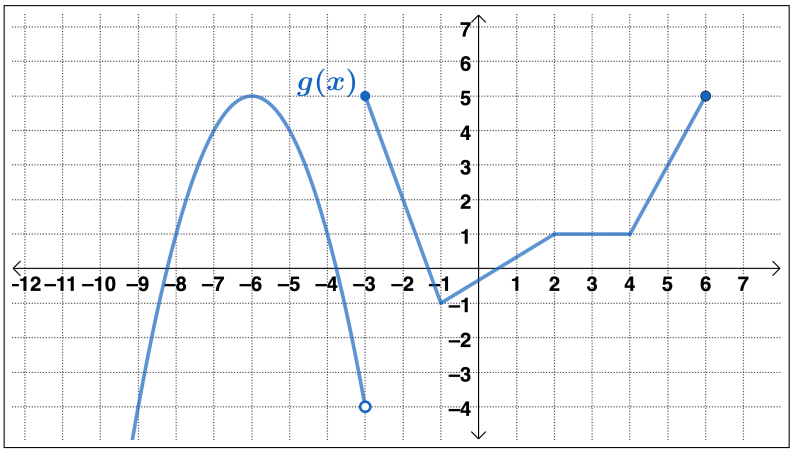
(d) $f(x) = \dfrac{5^x}{\sqrt[3]{x^2}}$

11. Find the x-value(s) where the graph of $f(x) = e^x(x^2 - 2x - 2)$ has a horizontal tangent line.

    (Note: In the interest of time, I'll provide this: $f'(x) = e^x(x^2 - 4)$. But you should use derivative rules to ensure you could find this derivative correctly. If you get stuck come to a math learning center help session or visit your instructor's office hours.)

12. Find the x-value(s) where $f(x) = \dfrac{-20x}{x+2}$ has an instantaneous rate of change of -10.

13. Use the table and graph below to find each of the following.



x	$f(x)$	$f'(x)$
-6	4	10
-5	2	9
-4	0	10
-3	-4	6
-2	9	0
-1	3	-1
0	9	-4
1	8	0
2	7	4
3	3	5
4	1	7
5	-1	3

(a) $h'(-6)$ if $h(x) = x^2 g(x)$

(b) $p'(0)$ if $p(x) = \dfrac{e^x g(x)}{f(x)}$.

(*Hint*: The line segment on the interval $(-1, 2)$ contains the points $(-1, -1)$ and $(2, 1)$. The line that contains these two points is $y = \dfrac{2}{3}x - \dfrac{1}{3}$. Be sure you can calculate the equation of this line on your own and use this to help solve this problem.)