Week in Review 4

Monday, February 19, 2024 5:25 PM

TEXAS A&M UNIVERSITY Mathematics

Math 140 - Spring 2021
Week In Review ♯4 - Feb. 19, 2021

Section 3.1: Setting Linear Programming Problems

- Always Define Your Variables
- Objective Function
- Constraints

Pr 1. Set up, but do not solve.

A housing contractor wants to develop a 42 acre tract of land. He has three types of houses: a small 3 bedroom, a large 3 bedroom and a 4 bedroom house. The small three bedroom house requires \$70,000 of capital for a profit of \$20,000, the large three bedroom house requires \$84,000 of capital for a profit of \$25,000, and the four bedroom house requires \$100,000 of capital for a profit of \$24,000. The small three bedroom needs 3000 labor hours, the large three bedroom needs 3500 labor hours, and the 4 bedroom house needs 3900 labor hours. There are currently 250,000 labor hours available. If the small three bedroom house is on half an acre, the large 3 bedroom house is on 0.75 acres, the four bedroom house is on 1.5 acres and the contractor has 6 million in capital, how many of each type should be built to maximize the profit?

capital

profit ✓

labor hours

= acres

no green box

↑ total profit

no green dashed line

Variables:

the number of

s := small 3-bedroom houses built.

ℓ := the number of large 3-bedroom houses built

f := the number of 4-bedroom houses built

P := the total profit

⋮

Objective: Maximize/ Minimize $P = 20000s + 25000\ell + 24000f$

Subject to:

$70000s + 84000\ell + 100000f \leq 6000000$ capital

$3000s + 3500\ell + 3900f \leq 250000$ labor hours

$.5s + .75\ell + 1.5f \leq 42$ acres

$s \geq 0,\ \ell \geq 0,\ f \geq 0$ non-negativity constraints

Your water bottle company sells bottles in 20 ounces (the Sprinkle), 30 ounces (the Storm), and 40 ounces (the Hurricane). The amount of glass (square yards), stainless steel(pounds), and plastic(pounds) used in making each model are given in the table.

	~~Glass~~ glass	Metal	~~Wood~~ plastic
Sprinkle	1	2	1
Storm	2	1	3
Hurricane	2	3	6

How many sprinkles we make is unknown

The profit for the Storm is $1, for the Hurricane is $2 and for the Sprinkle is $1. Due to certain agreements, the company can make at most 250 Sprinkle bottles. If the company has 300 square yards of glass, 600 pounds of stainless steel, 800 pounds of plastic, how many of each type of ~~umbrella~~ water bottle should be produced in order to maximize the profit?

$s \leq 250$

Variables

P = the total profit.

s = the number of sprinkles made and sold.

t = the number of storms made and sold.

h = the number of hurricanes made and sold.

Objective: Maximize $P = 1 \cdot s + 1 \cdot t + 2h$

Subject to:

$0 \leq s \leq 250$

$s + 2t + 2h \leq 300$ glass

$2s + t + 3h \leq 600$ Steel

$s + 3t + 6h \leq 800$ plastic

$(s \geq 0)$ $t \geq 0$, $h \geq 0$

optional

You have \$15,000 to invest, some in Stock A and some in Stock B. You have decided that the money invested in Stock A must be at least twice as much as that in Stock B. However, the money invested in Stock A must not be greater than \$8,000. If Stock A earn 4% annual interest, and Stock B earn 5% annual interest, how much money should you invest in each to maximize your annual interest?

A = number of shares of stock A

B = number of shares of stock B

I = total annual interest

4% = .04

maximize $I = .04A + .05B$

subject to:

A not greater than 8000?

$A \geq 2B$

$A \leq 8000$

$A + B \leq 15000$ (total investment)

$A \geq 0, \quad B \geq 0$

$A \geq 2B$

$A \leq 8000$

$A + B \leq 15000$

+ solve

$A = 8000$

$A + B = 15000$

$8000 + B = 15000$

$B = 7000$

Three corner points

	$I = .04A + .05B$
★ (8000, 4000)	\$520 $.04 \cdot 8000 + .05 \cdot 4000$ = $320 + 200 = 520$
(0,0)	0
(8000, 0)	\$320 $.04 \times 8000$

Maximum is with \$8000 in stock A, \$4000 in stock B, annual interest of \$520

left overs investing \$15000 but only spent $8000 + 4000 = 12000$

left-over = $15000 - 12000 = 3000$

$\boxed{\$3000 \text{ left over}}$

An independent taffy company makes three flavors of taffy: strawberry, lemon, and orange. Each strawberry taffy requires 4 minutes to cool and 1 minute to wrap in paper. Each orange taffy requires 3 minutes to cool and 1.5 minutes to wrap in paper. Each lemon taffy requires 4 minutes to cool and 2 minutes to wrap in paper. There are a total of 1.5 hours available for cooling and 0.5 hours available for wrapping. Determine the production of each taffy to maximize profit if the profit on the sale of each orange, lemon, and strawberry taffy is 75 cents, 60 cents, and 50 cents respectively, and previous sales indicate that they should produce at least three times as many strawberry taffy as lemon taffy. How many of each flavor should the company make to maximize their profits? What is the maximum profit and is any time leftover in cooling or wrapping?

s = the number of strawberry taffies produced and sold

l = the no. of lemon taffies produced and sold

r = the number of orange taffies produced and sold

P = the total profit.

Maximize $P = .50s + .60l + .75r$

subject to:

~~$4s + 4l + 3r \leq 1.5$ hrs~~

deduct points cooling

are not minutes!

~~4s + 4l + 3r ≤ 1.5 hrs~~

are not minutes!

1) $4s + 4l + 3r \le 1.5 \cdot 60 = 90$ (cooling)

≤ 90

$1s + 2l + 1.5r \le 30$ (wrapping)

$s \ge 3l$ or $s - 3l \ge 0$ .5 hrs

$s \ge 0$ $l \ge 0$ $r \ge 0$

(three times as many straw. as lemon)

$3s \le l$ or $s \le 3l$ l lemons → $3l$ strawbers.

$\ge$ $s \ge 3l$ $s = 3l$

$\le$ at most

$\ge$ at least

Section 3.2: Graphing Systems of Linear Inequalities in Two Variables

- Solution set to a linear inequality is half of the coordinate plane, while the solution set to a system of linear inequalities is the region of points that satisfy **all** of the linear inequalities in the system.
- Boundary Line - the corresponding linear equation for a a linear inequality ← where = is true
- True Shading vs. Reverse Shading
- Unbounded vs. Bounded solution sets
- Corner Points

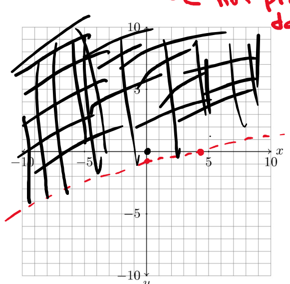
$f(x) \le a$ true shading vs. true shading $f(x) \ge a$

Pr 1. Graph the inequality $5x - 9y < 21$, labeling the boundary line and the solution set with **S**.

= not present dashed

$5x - 9y < 21$

True shading



$5x - 9y = 21$

x-intercept $y = 0$

$5x - 9 \cdot 0 = 21$

$5x = 21$

$x = \frac{21}{5} \approx 4.2$ $(4.2, 0)$

y-intercept $x = 0$

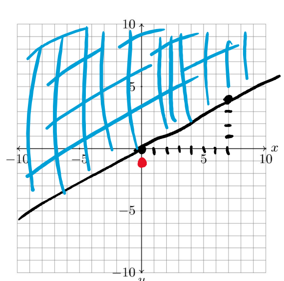
$5 \cdot 0 - 9y = 21$

$-9y = 21 \rightarrow y = \frac{21}{-9} = -\frac{7}{9}$

Test point $(0,0)$

$5 \cdot 0 - 9 \cdot 0 = 0 < 21$

Pr 2. Graph the inequality $-4x + 7y \ge 0$, labeling the boundary line and the solution set with **S**.



x-intercept

$y = 0$ $-4x + 7 \cdot 0 = 0$

$-4x = 0$

$x = 0$

$-4x + 7y = 0 \rightarrow 7y = 4x$

$y = \frac{4}{7}x$

Test point $(0, -1)$:

$-4 \cdot 0 + 7 \cdot (-1) = 0 - 7 = -7 < 0$

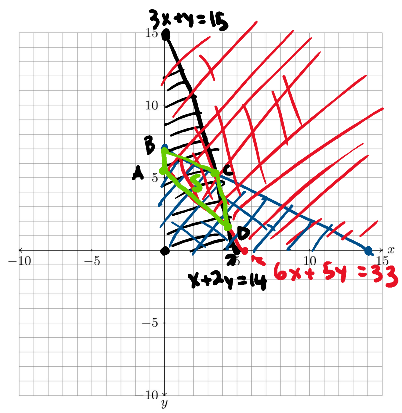
need ≥ 0

$\begin{cases} y \ge mx + b \\ y \le mx + b \end{cases}$

bounded or unbounded and the exact corner point(s) of the solution set.

- $3x + y \le 15$ → $y \le -3x + 15$
  $6x + 5y \ge 33$
  $x + 2y \le 15$
  $x \ge 0, y \ge 0$

	$3x + y \le \cancel{12}$ 15	$6x + 5y \ge \cancel{20}$ 33	$x + 2y \le \cancel{14}$ 15	$x \ge 0$	$y \ge 0$
Boundary Line:	$3x+y=15$	$6x+5y=33$	$x+2y=14$	$x=y=0$	
x-intercept: $y=0$	$\frac{3x}{3}=\frac{15}{3}$ → $x=5$ $(5,0)$	$6x+5\cdot0=33$, $x=\frac{33}{6}=\frac{11}{2}=5.5$	$x+2\cdot0=14$, $x=14$		
y-intercept: $x=0$	$3\cdot0+y=15$, $y=15$	$6\cdot0+5y=33$, $y=\frac{33}{5}=6.6$	$0+\frac{2y}{2}=\frac{14}{2}$, $y=7$		
Test Point: $(0,0)$	$0 \le 15$ ✓	$0 \ge 33$ X	$0 \le 15$ ✓		



Corner Points: A $x=0$ red line; B $x=0$ blue line; C black blue; D black red

A intercept $(0, 6.6)$

A=

B $= (0,7)$ intercept

C $3x+y=15$
$x+2y=14$ → $x=-2y+14$ + substitute

$\begin{bmatrix} 3 & 1 & | & 15 \\ 1 & 2 & | & 14 \end{bmatrix}$ → use calc to solve

$3(-2y+14)+y=15$
$-6y+42+y=15$
$-5y+42=15$
$-5y=-27$
$y=+\frac{27}{5}=+5.4$
$x=-2y+14$
$=-10.8+14$
$=3.2$
$C=(3.2, 5.4)$

-42
$+15$
27

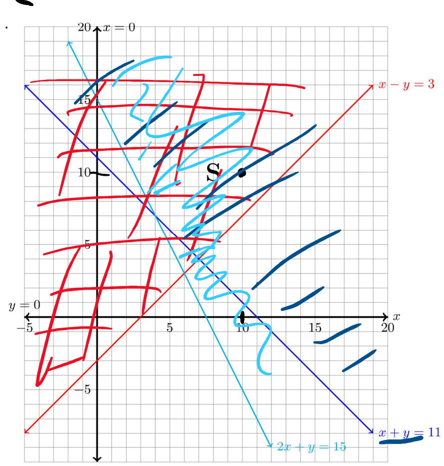
D $6x+5y=33$
$3x+y=15$

$\begin{bmatrix} 6 & 5 & | & 33 \\ 3 & 1 & | & 15 \end{bmatrix}$

rref

$\begin{bmatrix} 1 & 0 & | & 14/3 \\ 0 & 1 & | & 1 \end{bmatrix}$

$(\frac{14}{3}, 1)$



$x+y \ge 11$ → Test point $(10,10)$ $10+10 \ge 11$
$2x+y \ge 15$ → $2\cdot10+10 \ge 15$
$x-y \le 3$ → $10-10 \le 3$

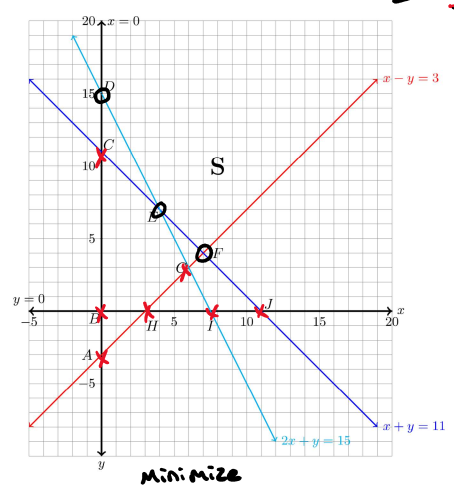
Section 3.3: Graphical Solution of Linear Programming Problems

- Feasible Region
- Know the parts of the Fundamental Theorem of Linear Programming
- Method of Corners
  - Set up a linear programming problem algebraically. Section 3.1
  - Graph the constraints and determine the feasible region. Section 3.2
  - Identify the exact coordinates of all corner points of the feasible region.
  - Determine whether or not the linear programming problem will have a solution. check conditions
  - If a solution will exist, evaluate the objective function at each corner point and determine the optimal point.
- Leftovers

over the region, if they exist and where they occur.

maximize and minimize

$Z = 3x + y$



$x + y \geq 11$

$2x + y \geq 15$

$x - y \leq 3$

S is unboundend in first quadrant

So no maximum exists.

but the minimum exists at a corner point or "boundary edge".

Minimize

(x, y)	$Z = 3x + y$	
A: $(0, -3)$		
B: $(0, 0)$		
C: $(0, 11)$		
D: $(0, 15)$	$3 \cdot 0 + 15 = 15$ ← smallest	
E: $(4, 7)$	$3 \cdot 4 + 7 = 12 + 7 = 19$	
F: $(7, 4)$	$3 \cdot 7 + 4 = 21 + 4 = 25$	
G: $(6, 3)$		
H: $(3, 0)$		
I: $(7.5, 0)$		
J: $(11, 0)$		

minimum is at $(0, 15)$ with a minimum value of 15.

No maximum exists.

Minimize

Objective: ~~Maximize~~ $P = 12x + 8y$

Subject to: $3x + y \leq 15$

$6x + 5y \geq 33$

$x + 2y \leq 15$

$x \geq 0,\ y \geq 0$

our problem from earlier

A (0, 6.6)

B (0, 7)

C (3.2, 5.4)

D $\left(\frac{14}{3}, 1\right)$

Minimize / Maximize

$12x + 8y$

(0, 6.6)	$12 \cdot 0 + 8 \cdot 6.6 = 52.8$ ☆
(0, 7)	$12 \cdot 0 + 8 \cdot 7 = 56$
(3.2, 5.4)	$12 \cdot 3.2 + 8 \cdot 5.4 = 81.6$ ☆
$\left(\frac{14}{3}, 1\right)$	$12 \cdot \frac{14}{3} + 8 \cdot 1 = 64$

minimum is at (0, 6.6) with a value of 52.8

Maximum is at (3.2, 5.4) with a value of 81.6.

Taffy problem

to cool and 1.5 minutes to wrap in paper. Each lemon taffy requires 4 minutes to cool and 2 minutes to wrap in paper. There are a total of 1.5 hours available for cooling and 0.5 hours available for wrapping. Determine the production of each taffy to maximize profit if the profit on the sale of each orange, lemon, and strawberry taffy is 75 cents, 60 cents, and 50 cents, respectively, and previous sales indicate that they should produce at least three times as many strawberry taffy as lemon taffy. How many of each flavor should the company make to maximize their profits? What is the maximum profit and is any time leftover in cooling or wrapping?

Maximize $P = .5s + .6l + .75r$

Subject to

$4s + 4l + 3r \le 90$ (cool)

$s + 2l + 1.5r \le 30$ (wrap)

$s \ge 3l$

$s \ge 0 \quad l \ge 0 \quad r \ge 0$

Three variables ....

- Can you graph this system?
  not how we have been doing it...
- Section 3.4

There is a feasible region with corner points..

problem: computing corners gets harder....

next week.