Math 151 - Hands On, Grades Up 6 (3.8-3.9, Exam 2 Review)

Justin Cantu

Please scan the QR code below.

We will begin at 7PM. A problem will be displayed on the table monitors. Collaborate with your table on how to solve each problem. If you have a question, raise your hand. After several minutes, the solutions will be displayed on the wall monitors. Feel free to take a picture of the solution, as the solutions are not posted.

1. A bird population grows at a rate proportional to its size. There are 1000 birds initially and 3000 birds after 2 years. When will the population reach 5000 birds?

2. The height of a triangle is decreasing at a rate of 2 cm/s and the length of the base is increasing at a rate of 4 cm/s. When the height is 3 cm and the base is 8 cm, how fast is the area of the triangle changing?

3. A camera is positioned 9 m away from a drone. The drone rises vertically at a speed of 5 m/s.

   (a) At what rate is the distance between the camera and the drone changing when the drone has risen 12 m?

(b) If the camera is always kept aimed at the drone, how fast is the camera's angle of elevation changing when the drone has risen 12 m?

4. Find $y' = \dfrac{dy}{dx}$.

   (a) $x^2y^2 - \tan(y) = 4x + 5y + 2^{10}$

   (b) $e^{xy} = 4x^3 + \sec(3y)$

5. If $f(x) = \ln(\cos x - \sin x)$, find $f'(0)$.

6. Find $y' = \dfrac{dy}{dx}$ (in terms of x).

   (a) $y = (2x + 4)^{\cot x}$

   (b) $y = (\csc x)^{x^2}$

7. Given that $g(5) = -3$, $g'(5) = 6$, $h(5) = 4$, $h'(5) = -2$, find $f'(5)$.

(a) $f(x) = g(x)e^{h(x)}$

(b) $f(x) = \dfrac{g(x)}{h(x) + 1}$

(c) $f(x) = h(g(x))$ [given $h'(-3) = 2$ and $h'(6) = 10$]

8. Find $h''(1)$ if $h(x) = \arctan(x^2)$.

9. Find the t-value(s) where the tangent line to the curve $x = t^3 - 3t^2$, $y = \frac{2}{3}t^3 + t^2 - 4t$ is vertical or horizontal.

10. Find an equation of the tangent line to the curve parametrized by $x = \sin\left(\frac{\pi}{6}t\right)$, $y = e^{2-t}$ at the point where $t = 2$.

11. Find the slope of the tangent line to the curve $x = y^2 + y$ at the point $(2, 1)$.

12. The position of an object is given by $\mathbf{r}(t) = \langle \sin(3t), 2\cos(3t) \rangle$. Find the velocity and speed of the object at time $t = \frac{\pi}{3}$.

13. Find a tangent vector to the curve $\mathbf{r}(t) = (t^2 - t - 1)\mathbf{i} + \tan\left(\frac{\pi t}{4}\right)\mathbf{j}$ at the point $(-1, 1)$.

14. An object is thrown vertically upward from the ground and its height after t seconds is given by $h(t) = 64t - 16t^2$. What is the maximum height reached by the object?

15. Determine the x-value(s) in the interval $[0, 2\pi)$ where the graph of $f(x) = \cos^2(x) + \cos(x)$ has a horizontal tangent line.

16. Differentiate the following functions.

    (a) $f(x) = 2e^{3x} + x^e - \pi$

    (b) $h(t) = 3t\cos(t^3 + 3^t)$

(c) $y = \dfrac{\sqrt{x} + e^{\pi}}{\tan x}$

(d) $f(x) = \log_9(\sec x)$

(e) $g(u) = \ln(ue^{-2u})$

(f) $f(x) = \sqrt{1 - x^2} \arccos x$