

1. Let $f(x, y) = \sin(xy) + \pi$. Find $f(1, \pi/2)$.

2. Let $f(x, y, z) = y + xz$. Find $f(-3, 2, 1)$.

3. Find the domain and range of the functions:

(a) $f(x, y) = \ln(2 - x^2 - y^2)$;

(b) $f(x, y, z) = e^{-\frac{1}{x^2+y^2+z^2}}$

4. Sketch the graphs of the functions:

(a) $f(x, y) = x^2 + y^2 - 3$

(b) $f(x, y) = \sqrt{4 - x^2 - y^2}$

(c) $f(x, y) = -2x - 4y + 4$

5. Classify all level curves of the functions

(a) $f(x, y) = \sqrt{x - y}$

(b) $f(x, y) = e^{-x^2 - y^2}$

6. Describe level surfaces of the function $f(x, y, z) = -x^2 - y^2 - z^2$

7. Find the first partial derivatives of the functions

(a) $f(x, y) = x^4 + 5xy^3$

(b) $f(x, y) = y^2 \cos(xy)$

(c) $f(x, y) = x^y$

(d) $f(x, y, z) = xy^2 e^{-xz}$

(e) $f(u, v, w) = \sqrt{u^4 + v^2 \cos w}$

8. Find all second-order derivatives of the function

(a) $z = xe^{-2y}$

(b) $v = r \cos(x + 2t)$

9. Where does the plane tangent to the surface $z = e^{x-y}$ at $(1, 1, 1)$ meet the z -axis?

10. Show that the surfaces given by $f(x, y) = x^2 + y^2$ and $g(x, y) = -x^2 - y^2 + xy^3$ have the same tangent plane at $(0, 0)$.

11. Find the differential of the function

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

12. Use differentials to estimate

$$\sqrt{(4.01)^2 + (3.98)^2 + (2.02)^2}$$

13. The two legs of a right triangle are measured as 5 m and 12 m respectively, with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the maximum error in the calculated value of the area of the triangle.

14. If $z = y + f(x^2 - y^2)$, where f is differentiable, show that

$$y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x$$

15. Let

$$w = \cos xy + y \cos x,$$

where

$$x = e^{-t} + 3s, \quad y = 5e^{2t} - \sqrt{s}$$

Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$.

16. If

$$yz^4 + xz^3 = e^{xyz}$$

find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.