# 1 Week 15 HOGU: 6.2, Final Exam Review Part 2

**Problem 1.** The John Weeks Enterprise is booming! The business needs to save up $\$150,000$ for a planned expansion. They make an initial deposit of $\$25,000$ and plan on depositing \$500 at the end of each month in T-bills, which currently yield 5.16% APR. Say that interest in these T-bills is compounded monthly. How many years will it take the John Weeks Enterprise to save up the $\$150,000$ they need?

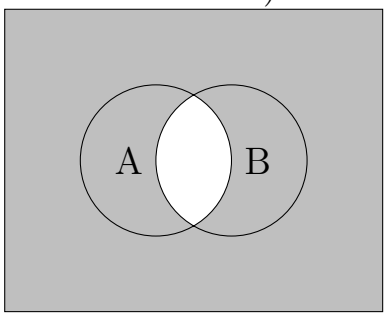
**Problem 2.** You are 18 years of age and want to be ready to retire at age 65! To supplement your employer's retirement fund for you, you put $\$1,000$ in a stable mutual fund that has a 3% APR. You continue investing \$300 in that mutual fund every month from now until you turn 65. If interest is compounded monthly, how much money do you have to retire on at age 65?

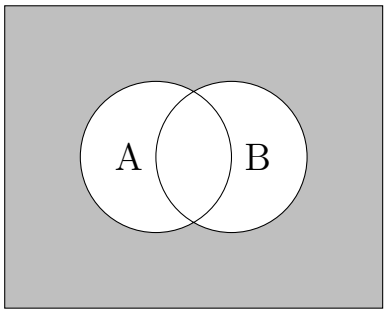
**Problem 3.** You are building the boat "Sailing for Weeks", which will currently cost $\$44,750$ to craft. You make a down payment of $\$10,000$ and finance the rest with a 10-year loan. The loan charges 7% interest, compounded quarterly.

(a) After making quarterly payments on the boat for 3 years, how much of the principal of your loan is still unpaid?

(b) After 3 years on your old loan above, you find a loan company that will refinance your loan at an APR of 5%, compounded quarterly, with a payment schedule that has you paying off the loan in 5 years! You decide to refinance your purchase and to make quarterly payments on the new loan. How much will your quarterly payments be?

**Problem 4.** Using union, intersection, and complements, how would you describe the shaded-in regions of these Venn diagram? (There is more than one correct answer.)



**Problem 5.** The following is a probability distribution with a missing entry:

| Outcome | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Probability | $\frac{18}{100}$ | $\frac{23}{100}$ | | $\frac{9}{100}$ | $\frac{33}{100}$ |

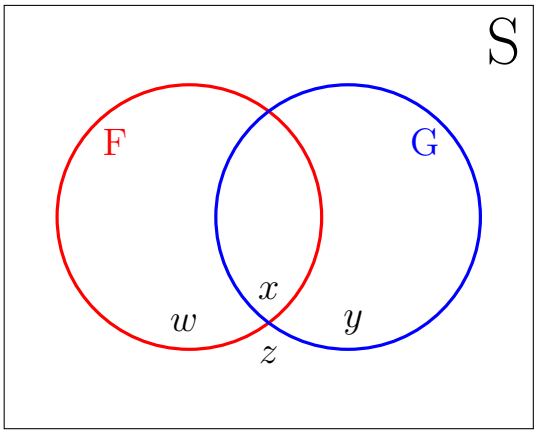
Let $A = \{1, 3, 5\}$ and let $B = \{2, 4, 5\}$. What is $P\left(A \cup B^C\right)$?

**Problem 6.** Given that $F$ and $G$ are two events of an experiment with $P(F) = 0.4$, $P(G) = 0.5$, and $P(F \cap G) = 0.2$, calculate the following probabilities:

(a) $P(F^C)$

(b) $P(F \cup G)$

(c) Now say that $P(F) = 0.4$, $P(G) = 0.5$, and $P(F \cup G^C) = 0.8$. Use the Venn Diagram below to find $P(F \cap G)$.

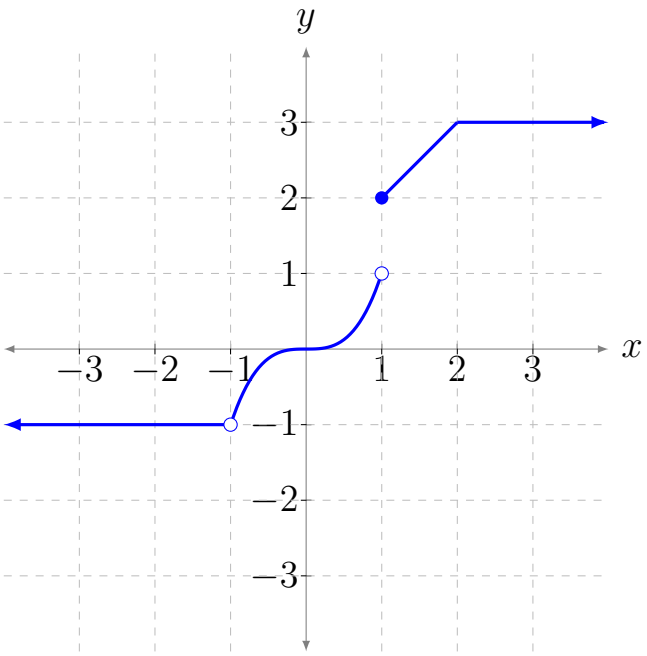


**Problem 7.** A local group is sponsoring a game at the Renaissance Fair! A foolish jester asks you to pay \$1 to play the game, then flips a fair two-sided coin. As it is in the air, the player calls "heads" or "tails". If the coin lands on the side the player called out, they win \$5! Otherwise, the player wins nothing.

(a) Let $X$ be the amount of *net winnings*, in dollars, that a player makes from playing this game. Write a probability distribution for $X$.

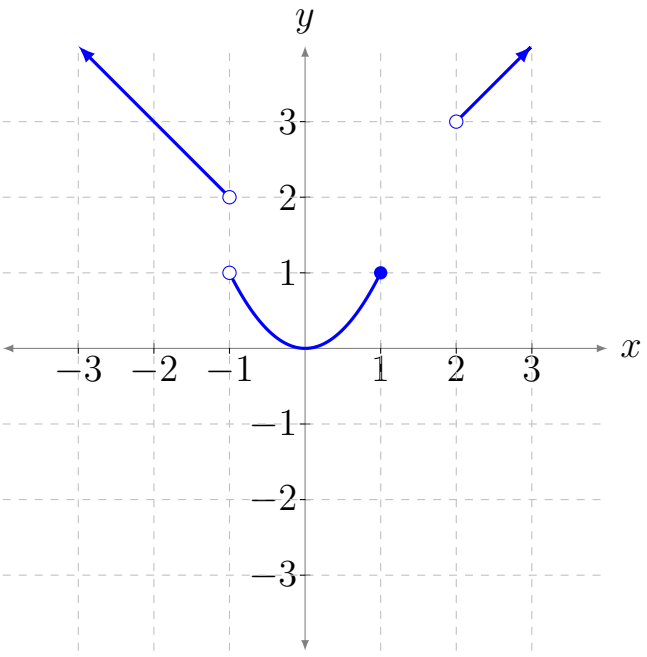
(b) What is the expected amount of net winnings for the player? Is this a fair game?

**Problem 8.** Find the domain and range of the following functions.




**Domain:**

**Range:**




**Domain:**

**Range:**

**Problem 9.** Compute the domain of the following functions. Then state all holes and vertical asymptotes of the function. (Remember that, for holes, you must give an $x$- and $y$-coordinate. Write your answer as $(x, y)$.)

(a) $f(x) = \dfrac{(x+3)^2}{(x-1)^2(x+3)}$

(b) $f(x) = \dfrac{(x+3)}{(x-1)^2(x+3)^2}$

**Problem 10.** Compute and completely simplify the difference quotient for the function $f(x) = \sqrt{x}$.

(a) What is $f(x+h) - f(x)$?

(b) Write your answer from above over 1, like $\dfrac{f(x+h)-f(x)}{1}$, then *rationalize the numerator*. Simplify your answer.

(c) Using your answers from above, calculate $\frac{f(x+h)-f(x)}{h}$, the difference quotient of $f(x) = \sqrt{x}$.

**Problem 11.** Find the domain of the following function:

$$f(x) = \begin{cases} \ln(-x) & \text{if } x \leq 2 \\ \dfrac{9}{\sqrt{4-x}} & \text{if } x > 3 \end{cases}$$

**Problem 12.** Solve the equation $e^{4x} = 5e^{7x}$ for $x$.

**Problem 13.** Write the expression $\frac{1}{3}\ln(x) - \ln(x+y) + 4\ln(2z)$ as a single logarithm. Assume all variables represent positive numbers.