

Section 11.10 Maclaurin Series to Memorize

Have these Maclaurin series memorized, and know when to use them. The following Maclaurin Series may be used, without proof, in order to achieve a higher goal.

$$1. \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ with radius of convergence } R = 1$$

$$2. e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ with radius of convergence } R = \infty$$

$$3. \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ with radius of convergence } R = \infty$$

$$4. \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ with radius of convergence } R = \infty$$

$$5. \arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ with radius of convergence } R = 1$$

To be clear, with enough practice, you will learn how to manipulate the series above to achieve a different result. A few illustrations follow:

(a) To express $g(x) = x^3 \sin(x^4)$ as a Maclaurin Series, we can state $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ without proof, and then manipulate this result to express $g(x)$ as a Maclaurin Series.

(b) To express $g(x) = \ln(4+x^3)$ as a Maclaurin Series, we can state $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ without proof, then manipulate this result to express $g(x)$ as a Maclaurin Series.

(c) To express $g(x) = \frac{x^3}{(1-4x)^2}$ as a Maclaurin Series, we can state $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ without proof, then manipulate this result to express $g(x)$ as a Maclaurin Series.