WIR#10

MATHEMATICS
TEXAS A&M UNIVERSITY

Week-in-Review#10 - Exam 3 Review
Covers Sections 3.4-3.5, 4.1-4.4
Math 142 - Fall 2024, by K. Kilmer

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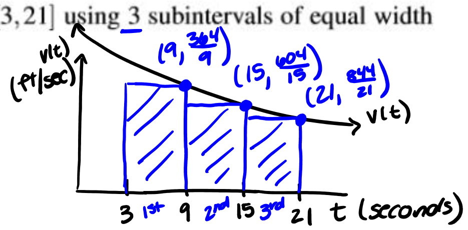
1. Particular values of a velocity function, $v(t)$, (in feet/second) are given in the table below, where t is in seconds.

t	3	6	9	12	15	18	21	24
$v(t)$	$\frac{124}{3}$	$\frac{122}{3}$	$\frac{364}{9}$	$\frac{121}{3}$	$\frac{604}{15}$	$\frac{362}{9}$	$\frac{844}{21}$	$\frac{241}{6}$

$\Delta t = \frac{b-a}{n} = \frac{21-3}{3} = 6$

Estimate the total distance traveled by the object on the interval $[3, 21]$ using 3 subintervals of equal width and a right-hand Riemann sum.

a b n



Total Distance Traveled ≈ $R_3 = 6\cdot v(9) + 6\cdot v(15) + 6\cdot v(21)$

$= 6\left(\frac{364}{9}\right) + 6\left(\frac{604}{15}\right) + 6\left(\frac{844}{21}\right)$

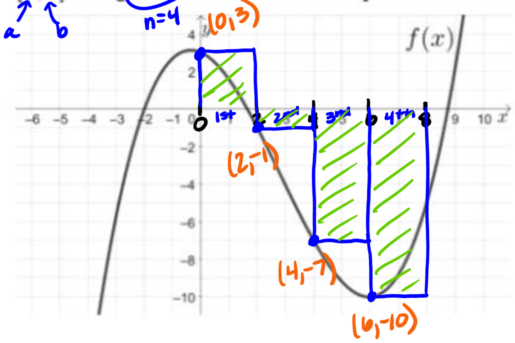
$= \boxed{\frac{76168}{105}\text{ feet}}$

$\cancel{\text{seconds}}\left(\frac{\text{feet}}{\cancel{\text{second}}}\right) = \text{feet}$

2. Given the graph of $f(x)$ below, estimate the net area between the graph of $f(x)$ and the x-axis on the interval $[0, 8]$ using four subintervals of equal width and a left-hand Riemann sum.

a b n=4

$\Delta x = \frac{b-a}{n} = \frac{8-0}{4} = 2$



Net Area ≈ $L_4 = 2\cdot f(0) + 2\cdot f(2) + 2\cdot f(4) + 2\cdot f(6)$

$= 2(3)+2(-1)+2(-7)+2(-10)$

$= 6-2-14-20$

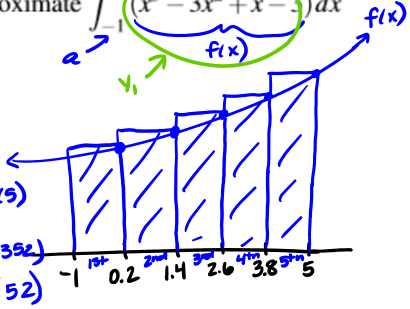
$= \boxed{-30}$

3. Use a Riemann sum with $\overset{n}{5}$ equal subintervals and right endpoints to approximate $\int_{a=-1}^{b=5} \underbrace{(x^3-3x^2+x-3)}_{f(x)}\,dx$

Y_1

This is not drawn accurately

$\Delta x = \frac{b-a}{n} = \frac{5-(-1)}{5} = 1.2$

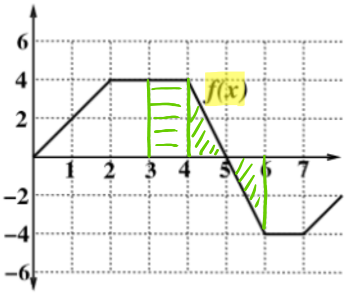


$$\int_{-1}^{5}(x^3-3x^2+x-3)dx \approx R_5 = 1.2\cdot f(0.2) + 1.2\cdot f(1.4) + 1.2\cdot f(2.6) + 1.2\cdot f(3.8) + 1.2\cdot f(5)$$

$$= 1.2(-2.912) + 1.2(-4.736) + 1.2(-3.104) + 1.2(12.352) + 1.2(52)$$

$$= \boxed{64.32}$$

4. Use the figure below to evaluate the following integral:



$$\int_3^6 \underline{(x^2+2f(x))}\,dx = \overset{A}{\boxed{\int_3^6 x^2 dx}} + 2\overset{B}{\boxed{\int_3^6 f(x)dx}}$$

$$= (63) + 2(4)$$

$$= 63 + 8 = \boxed{71}$$

A) $\int_3^6 x^2 dx = \frac{1}{3}x^3\Big|_3^6 = \frac{1}{3}(6)^3 - \frac{1}{3}(3)^3 = \underline{63}$

Recall,

$\int_a^b \underline{f(x)}dx = F(x)\Big|_a^b$
$= F(b) - F(a)$

B) $\int_3^6 f(x)dx = (1)(4) + \frac{1}{2}(1)(4) - \frac{1}{2}(1)(4)$
$= \underline{4}$

① $\int (\overset{u}{\text{function}})^n \cdot \underline{\qquad}$

② $\int \frac{1}{\text{function}} \cdot \underline{\qquad}$

③ $\int b^{\text{function}} \cdot \underline{\qquad}$

$\boxed{u = \underline{x^5-2x+8},}$

$du = \underline{(5x^4-2)dx}$

5. Evaluate the following integral:

$$\int_k^4 \frac{15x^4-6}{x^5-2x+8}\,dx = \int_k^4 \frac{1}{\underbrace{x^5-2x+8}_{u}}(15x^4-6)\,dx$$

$$= \int_{x=k}^{x=4} \frac{1}{\underbrace{x^5-2x+8}_{u}} \cdot 3\underbrace{(5x^4-2)dx}_{du}$$

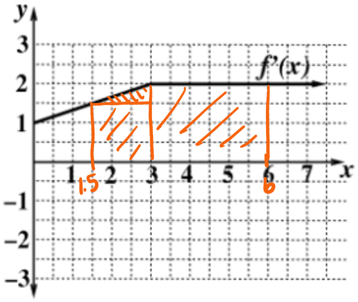
$$= \int_{k^5-2k+8}^{1024} \frac{1}{u}\cdot 3du = 3\int_{k^5-2k+8}^{1024} \frac{1}{u}du$$

$$= 3\cdot \ln|u| \Big|_{k^5-2k+8}^{1024} = \boxed{3\ln|1024| - 3\ln|k^5-2k+8|}$$

When doing u-substitution on a definite integral, we must change our limits of int. to be in terms of u:

- when $x=k$: $u = \underline{k^5-2k+8}$
- when $x=4$: $u = 4^5-2(4)+8 = \underline{1024}$

6. Use the graph of $f'(x)$ below and the fact that $\overset{a}{f(1.5)} = 9$ to find $\overset{b}{f(6)}$.



Recall,
$$\int_a^b f'(x)dx = f(x)\Big|_a^b = f(b)-f(a)$$

$$\int_{1.5}^{6} f'(x)dx = f(6) - \overset{9}{f(1.5)}$$

$$\int_{1.5}^{6} f'(x)dx = (1.5)(1.5)+\tfrac{1}{2}(1.5)(.5)+3(2)$$
$$= 8.625$$

$$8.625 = f(6) - 9$$
$$+9 \qquad\qquad +9$$
$$\boxed{17.625 = f(6)}$$

7. The marginal revenue function for a product is given to be $R'(x) = 20-0.2x$ dollars per item when x items are sold. Find the change in revenue when the number of items sold changes from 20 to 45.

Change in revenue from $x=20$ to $x=45$ $= \int_{20}^{45} R'(x)dx = \int_{20}^{45}(20-0.2x)dx$

$\approx \text{fnInt}(20-0.2x, x, 20, 45)$

$$= \int_{\boxed{20}}^{\boxed{45}} \boxed{20-0.2x}\, d\,\boxed{x} = \boxed{\$337.50}$$

8. Find the absolute extrema of $f(x) = \boxed{\dfrac{x^2-21}{x+5}}$ on the interval $[-4,1]$.

Y_1

Domain: $x \neq -5$

We have a function that is continuous on a closed int $\Rightarrow$ Use closed Int. method.

$$f'(x) = \frac{(x+5)(2x)-(x^2-21)(1)}{(x+5)^2} = \frac{2x^2+10x-x^2+21}{(x+5)^2}$$

$$= \frac{x^2+10x+21}{(x+5)^2} = \frac{(x+7)(x+3)}{(x+5)^2}$$

$f'(x)=0$ when $(x+7)(x+3)=0$

$x+7=0$ $\quad$ $x+3=0$

$\cancel{x=-7}$ $\quad$ $\boxed{x=-3}$ ← only CV on our interval

$f'(x)$ DNE when $(x+5)^2=0$

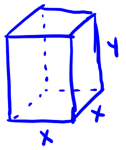
$x+5=0$

$\cancel{x=-5}$

x	$f(x)$
-4	-5
-3	-6
1	$-10/3 \approx -3.33$

$\boxed{\begin{array}{l}\text{Abs Max of } -10/3 \text{ at } x=1.\\ \text{Abs Min of } -6 \text{ at } x=-3\end{array}}$

9. A box with a square base and an open top is being constructed to have a volume of 80 m^3. The material for the base costs \$5 per square meter and the material for the sides costs \$2 per square meter. Find the dimensions of the box that will minimize the cost of materials.



Known: $x^2y=80$

$\boxed{y=\dfrac{80}{x^2}}$

Objective: Minimize

Cost $= 5x^2+2(4xy)$

$= 5x^2+8xy$

$\boxed{\text{Minimize} \quad C(x)=5x^2+8x\left(\frac{80}{x^2}\right) \quad \text{on} \quad (0,\infty)}$

$5x^2+\dfrac{640\cancel{x}}{x^{\cancel{2}}} = 5x^2+\dfrac{640}{x}$

Interval:

$x>0$ $\quad$ $y>0$

$y=\dfrac{80}{x^2}$

y will always be positive when $x>0$.

Int: $(0,\infty)$

$C(x)=5x^2+640x^{-1}$

$C'(x)=10x-640x^{-2}$

$=10x-\dfrac{640}{x^2}$

$=\dfrac{10x^3}{x^2}-\dfrac{640}{x^2}$

$=\dfrac{10x^3-640}{x^2}$

$\dfrac{x^2 \cdot}{x^2 \cdot}\dfrac{10x}{1} - \dfrac{640}{x^2}$

$C'(x)=0$ when $10x^3-640=0$

$10x^3=640$

$\sqrt[3]{x^3}=\sqrt[3]{64}$

$x=4$ ← only CV on our interval

$C'(x)$ DNE when $x^2=0$

$\cancel{x=0}$ not on our interval

Since we have 1 CV on our int (that is not closed)
$\Rightarrow$ Use the FDT or the SDT. I will use SDT.

$C''(x)=10+1280x^{-3}$

$C''(4)=10+\dfrac{1280}{4^3}=+$ $\cup$ $\Rightarrow$ Abs min at $x=4$

$x=4\text{m}$

$y=\dfrac{80}{4^2}=5\text{m}$

$\boxed{\begin{array}{l}\text{The dimensions of } 4\text{m}\times 4\text{m}\times 5\text{m}\\ \text{will yield a minimum cost.}\end{array}}$

10. If $f'(x) = 3x^2 - 2e^x - \overset{x^{-5}}{\frac{1}{x^5}} - \frac{3}{x} - 2^x$ and $f(1) = \frac{5}{4}$, find $f(x)$.

① Find the general antiderivative

$$f(x) = \int f'(x)\,dx = \int \left(\underbrace{3x^2}_{}\underbrace{-2e^x}_{}\underbrace{-x^{-5}}_{}\underbrace{-3\cdot\frac{1}{x}}_{}\underbrace{-2^x}_{}\right)dx = 3\cdot\frac{1}{3}x^3 - 2e^x - \frac{1}{-4}x^{-4} - 3\cdot\ln|x| - \frac{1}{\ln 2}2^x + C$$

$$= x^3 - 2e^x + \frac{1}{4}x^{-4} - 3\ln|x| - \frac{1}{\ln 2}\cdot 2^x + C$$

② Use $f(1) = \frac{5}{4}$ to find C.

$$f(x) = x^3 - 2e^x + \frac{1}{4}x^{-4} - 3\ln|x| - \frac{1}{\ln 2}\cdot 2^x + C$$

$$\frac{5}{4} = (1)^3 - 2e^1 + \frac{1}{4}(1)^{-4} - 3\underbrace{\ln|1|}_{0} - \frac{1}{\ln 2}\cdot 2^1 + C$$

$$\frac{5}{4} = 1 - 2e + \frac{1}{4} - \frac{2}{\ln 2} + C$$

$-\frac{5}{4}$ $\quad$ $5/4$

$-5/4$

$$0 = -2e - \frac{2}{\ln 2} + C$$

$+2e + \frac{2}{\ln 2}$ $\quad$ $+2e$ $\quad$ $+\frac{2}{\ln 2}$

$$2e + \frac{2}{\ln 2} = C$$

③ Plug in C:

$$\boxed{f(x) = x^3 - 2e^x + \frac{1}{4}x^{-4} - 3\ln|x| - \frac{1}{\ln 2}\cdot 2^x + 2e + \frac{2}{\ln 2}}$$

11. Evaluate the following integral:

$$\int \frac{6(x+1)(7x-8)}{x^3}\,dx = \int \frac{6(7x^2 \overbrace{-8x+7x}^{-x} - 8)}{x^3} = \int \frac{42x^2 - 6x - 48}{x^3}\,dx = \int \left(\frac{42\cancel{x^2}}{x^{\cancel{3}}} - \frac{6\cancel{x}}{x^{\cancel{3}2}} - \frac{48}{x^3}\right)dx$$

$$= \int \left(\underbrace{42\cdot\frac{1}{x}}_{} - \underbrace{6x^{-2}}_{} - \underbrace{48x^{-3}}_{}\right)dx = 42\ln|x| - 6\cdot\frac{1}{-1}x^{-1} - 48\cdot\frac{1}{-2}x^{-2} + C$$

$$= \boxed{42\ln|x| + 6x^{-1} + 24x^{-2} + C}$$

12. The marginal profit function for a cell phone company is given by $P'(x) = \dfrac{2x^{5/3}}{\sqrt{x^{8/3}+513}} - 4$ dollars per cell phone, where x is the number of cell phones sold. If the break-even quantity for this company is 64 cell phones, find the company's profit when 216 cell phones are sold.

$\Rightarrow P(64)=0$

$\Rightarrow P(216)=?$

① Find the general antiderivative:

$$P(x) = \int P'(x)\,dx = \int\left(\frac{2x^{5/3}}{\sqrt{x^{8/3}+513}} - 4\right)dx = \underbrace{\int \frac{2x^{5/3}}{\sqrt{x^{8/3}+513}}\,dx}_{A} - \underbrace{\int 4\,dx}_{B}$$

Ⓐ

$$\int \Big(\underbrace{x^{8/3}+513}_{u}\Big)^{-1/2} \cdot 2\underbrace{x^{5/3}\,dx}_{\frac{3}{8}du}$$

$u = x^{8/3}+513$

$du = \frac{8}{3}x^{5/3}\,dx$

$\frac{3}{8}du = \underbrace{x^{5/3}\,dx}$

$$= \int u^{-1/2}\cdot 2\cdot \tfrac{3}{8}\,du = \tfrac{3}{4}\int u^{-1/2}\,du$$

$$= \tfrac{3}{4}\cdot 2u^{1/2} + C$$

$$= \tfrac{3}{2}(x^{8/3}+513)^{1/2} + C$$

Thus,

$$P(x) = \tfrac{3}{2}(x^{8/3}+513)^{1/2} - 4x + C$$

Ⓑ

$$\int 4\,dx$$

$$= 4x + C$$

② Use $P(64)=0$ to solve for C

$$0 = \tfrac{3}{2}(64^{8/3}+513)^{1/2} - 4(64) + C$$

$$0 = \tfrac{3}{2}(257) - 256 + C$$

$$0 = 385.5 - 256 + C$$

$$0 = 129.5 + C$$

$$-129.5 = C$$

③ Plug in C:

$$P(x) = \tfrac{3}{2}(x^{8/3}+513)^{1/2} - 4x - 129.5$$

④ Find $P(216)$

$$P(216) = \tfrac{3}{2}(216^{8/3}+513)^{1/2} - 4(216) - 129.5$$

$$\approx \boxed{\$950.80}$$