

**05**

**MATH 152**

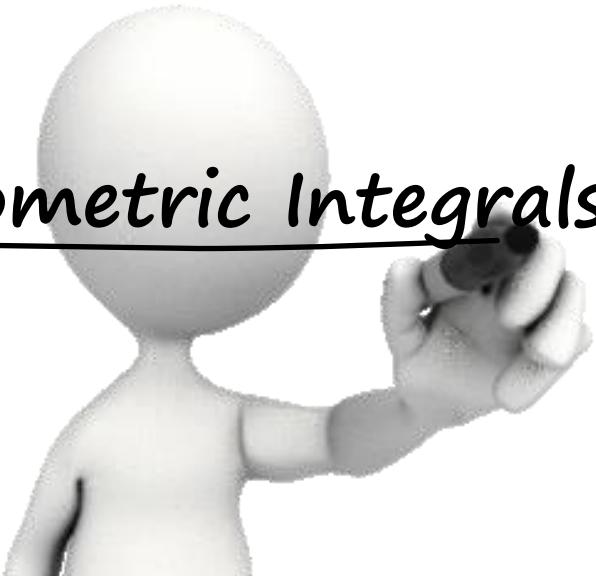
**Week in Review**

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**Trigonometric Integrals**

**Trigonometric Substitution**

**Integration by Partial Fractions**



## Trigonometric Integrals

Compute  $\int_0^{\pi/2} \sin^2(\theta) \cos^3(\theta) d\theta$ .

- (a)  $\frac{2}{5}$
- (b)  $\frac{4}{5}$
- (c)  $\frac{2}{15}$  ← correct
- (d)  $\frac{8}{5}$
- (e) None of the above

$$\begin{aligned}& \int \sin^2 \theta \cos^3 \theta d\theta \\&= \int \sin^2 \theta \cos^2 \theta (\cos \theta d\theta) \\&= \int \sin^2 \theta (1 - \sin^2 \theta)(\cos \theta d\theta) \\& u = \sin \theta \Rightarrow \\& \quad du = \cos \theta d\theta \\& \int_{x=0}^{x=\pi/2} \Rightarrow \\& \int_{u=0}^{u=1} \\& \int_0^1 u^2(1 - u^2)du \\&= \int_0^1 (u^2 - u^4)du \\&= \left[ \frac{1}{3}u^3 - \frac{1}{5}u^5 \right]_0^1 \\&= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}\end{aligned}$$

Which of the following is equal to  $\int_0^{\pi/4} \tan^2(\theta) \sec^4(\theta) d\theta$ ?

(a)  $\int_0^{\pi/4} u^2(u^2 - 1) du$

(b)  $\int_0^{\pi/4} u^2(1 + u^2) du$

(c)  $\int_0^{\sqrt{2}/2} u^2(1 + u^2) du$

(d)  $\int_0^1 u^2(u^2 - 1) du$

(e)  $\int_0^1 u^2(1 + u^2) du$  ← correct

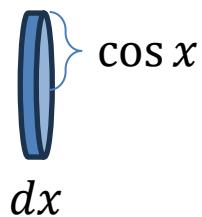
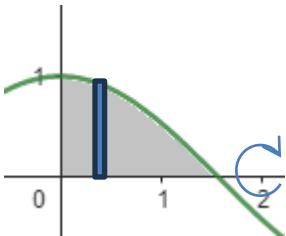
$$\begin{aligned} & \int_0^{\pi/4} \tan^2 \theta \sec^2 \theta (\sec^2 \theta d\theta) \\ &= \int_0^{\pi/4} \tan^2 \theta (\tan^2 \theta + 1)(\sec^2 \theta d\theta) \end{aligned}$$

$$\begin{aligned} u &= \tan \theta \\ &\Rightarrow du = \sec^2 \theta \end{aligned}$$

$$\begin{aligned} \int_{x=0}^{x=\pi/4} &\Rightarrow \\ &\int_{u=0}^{u=1} \\ &\int_0^1 u^2(u^2 + 1) du \end{aligned}$$

The region bounded by  $y = \cos x$  and the  $x$ -axis on the interval  $[0, \frac{\pi}{2}]$  is rotated about the  $x$ -axis.  
Find the volume of the resulting solid.

- (a)  $\frac{\pi}{4}$
- (b)  $\frac{\pi^2}{4}$  ← correct
- (c) 1
- (d)  $\frac{\pi^2}{2}$
- (e)  $\frac{\pi}{2}$



$$\begin{aligned}
 V(\text{shell}) &= \pi \cos^2 x \, dx \\
 V &= \int_0^{\pi/2} \pi \cos^2 x \, dx \\
 &= \int_0^{\pi/2} \pi \left[ \frac{1+\cos 2x}{2} \right] dx \\
 &= \frac{\pi}{2} \left[ x + \frac{1}{2} \sin 2x \right]_0^{\pi/2} \\
 &= \frac{\pi}{2} \left[ \frac{\pi}{2} \right] \\
 &= \frac{\pi^2}{4}
 \end{aligned}$$

# Exercise

Compute  $\int \cos^4(x) \sin^5(x) \, dx$

- (a)  $-\frac{1}{5} \cos^5(x) + \frac{1}{9} \cos^9(x) + C$
- (b)  $\frac{1}{6} \sin^6(x) - \frac{1}{4} \sin^8(x) + \frac{1}{10} \sin^{10}(x) + C$
- (c)  $\frac{1}{6} \sin^6(x) - \frac{1}{10} \sin^{10}(x) + C$
- (d)  $-\frac{1}{5} \cos^5(x) + \frac{2}{7} \cos^7(x) - \frac{1}{9} \cos^9(x) + C$
- (e) None of these.

Compute  $\int \cos^3(2x) \, dx$

- (a)  $-\sin(2x) + \frac{1}{3} \sin^3(2x) + C$
- (b)  $\frac{-1}{2} \sin(2x) + \frac{1}{6} \cos^3(2x) + C$
- (c) None of these.
- (d)  $\sin(2x) - \frac{1}{3} \sin^3(2x) + C$
- (e)  $\frac{1}{2} \sin(2x) - \frac{1}{6} \sin^3(2x) + C$

Evaluate  $\int \tan^3(x) \sec^5(x) \, dx$ .

- (a)  $\frac{1}{7} \tan^7 x - \frac{1}{5} \sec^5 x + C$
- (b)  $\frac{1}{7} \sec^7 x - \frac{1}{5} \tan^5 x + C$
- (c)  $\frac{1}{4} \sec^6 x - \frac{1}{6} \tan^{10} x + C$
- (d)  $\frac{1}{4} \sec^4 x - \frac{1}{6} \tan^6 x + C$
- (e)  $\frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x + C$

Compute  $\int \tan^3(x) \sec^3(x) \, dx$

- (a)  $\frac{1}{5} \sec^5(x) - \frac{1}{3} \sec^3(x) + C$
- (b)  $-\frac{1}{5} \sec^5(x) + \frac{1}{3} \sec^3(x) + C$
- (c)  $\frac{1}{5} \tan^5(x) - \frac{1}{3} \tan^3(x) + C$
- (d)  $-\frac{1}{5} \tan^5(x) + \frac{1}{3} \tan^3(x) + C$
- (e)  $-\sec^4(x) + \sec^2(x) + C$

Compute  $\int_0^{\pi/4} \sec^4(x) \, dx$

- (a)  $\frac{2}{3}$
- (b)  $\frac{32}{5}$
- (c)  $\frac{4}{3}$
- (d)  $\frac{4\sqrt{2}}{5}$
- (e)  $\frac{2\sqrt{2}-1}{3}$

Evaluate  $\int \sin^2(x) \, dx$ .

- (a)  $\frac{1}{2}x - \frac{1}{4} \sin(2x) + C$
- (b)  $\frac{1}{3} \sin x \cos x + C$
- (c)  $\frac{1}{2}x + \frac{1}{2} \sin x + C$
- (d)  $\frac{1}{2}x - \frac{1}{4} \sin x + C$
- (e)  $\frac{1}{3} \cos^3 x + C$

Compute  $\int \sin^7 \theta \cos^5 \theta \, d\theta$ .

Find  $\int \cos^4 x \, dx$

Compute  $\int \cos^2(x) \sin^2(x) \, dx$

# Exercise

Compute  $\int 2 \sin^2(2\theta) d\theta$

- (a)  $\theta - \frac{1}{2} \sin(2\theta) + C$
- (b)  $\theta - \frac{1}{4} \sin(4\theta) + C$
- (c)  $\theta + \frac{1}{2} \sin(2\theta) + C$
- (d)  $\theta + \frac{1}{4} \sin(4\theta) + C$
- (e) None of the above

Compute  $\int_0^{\pi/3} \tan^3(\theta) \sec(\theta) d\theta$

- (a)  $\frac{4}{3}$
- (b)  $\frac{16 - 9\sqrt{3}}{24}$
- (c)  $\frac{2}{3}$
- (d)  $\frac{-3\sqrt{3}}{8}$
- (e) None of the above

Compute  $\int_0^{\pi/2} \sin(2x) \cos x dx$ .

- (a)  $\frac{3}{2}$
- (b)  $\frac{2}{3}$
- (c) 0
- (d) 1
- (e)  $\frac{1}{2}$

$$\int_0^{\pi/2} \cos^3 x \sin^3 x dx =$$

$$\int \tan^4 x \sec^4 x dx =$$

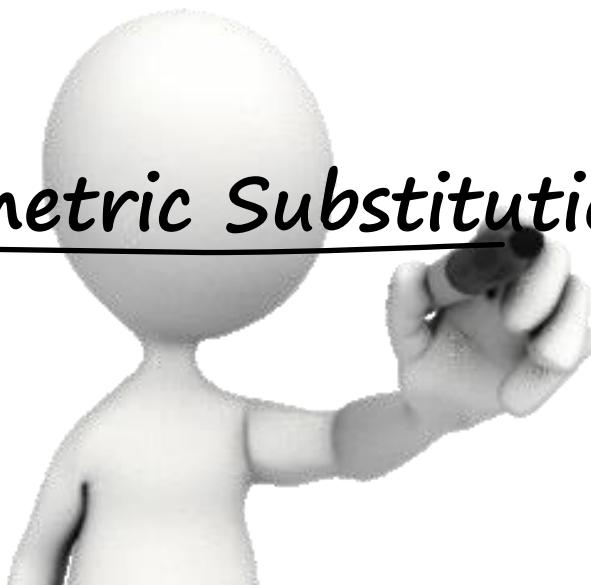
$$\int_0^{\pi/4} \sin^2(x) dx =$$

- (a)  $\frac{1}{12}$
- (b)  $\frac{2}{15}$
- (c)  $\frac{5}{12}$
- (d)  $\frac{-1}{12}$
- (e)  $\frac{-2}{15}$

- (a)  $\frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$
- (b)  $\frac{\tan^7 x}{7} - \frac{\tan^5 x}{5} + C$
- (c)  $\frac{\tan^9 x}{9} + \frac{\tan^5 x}{5} + C$
- (d)  $\frac{\tan^9 x}{9} - \frac{\tan^5 x}{5} + C$
- (e) None of these

- (a)  $\frac{\pi}{8} - \frac{1}{4}$
- (b)  $\frac{\pi}{8}$
- (c)  $\frac{\pi}{8} - \frac{1}{2}$
- (d)  $\frac{2}{\sqrt{2}} - \frac{\pi}{2\sqrt{2}}$
- (e)  $\frac{\pi}{8} + \frac{1}{4}$

## Trigonometric Substitution



After an appropriate substitution, the integral  $\int \sqrt{x^2 + x} dx$  is equivalent to which of the following?

- (a)  $\int \tan^2 \theta \sec \theta d\theta$
- (b)  $\frac{1}{4} \int \sec^3 \theta d\theta$
- (c)  $-\frac{1}{4} \int \sin^2 \theta d\theta$
- (d)  $\frac{1}{4} \int \tan^2 \theta \sec \theta d\theta$  ← correct
- (e)  $\int \cos^2 \theta d\theta$

$$\begin{aligned}& \int \sqrt{x^2 + x + \frac{1}{4} - \frac{1}{4}} dx \\&= \int \sqrt{\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}} dx \\&= \int \sqrt{\frac{1}{4}[(2x - 1)^2 - 1]} dx \\&\quad (2x - 1) = \sec \theta \\&\quad 2dx = \sec \theta \tan \theta d\theta \\&= \frac{1}{2} \int \sqrt{(\sec^2 \theta - 1)} \left( \frac{1}{2} \sec \theta \tan \theta d\theta \right) \\&= \frac{1}{4} \int \sqrt{\tan^2 \theta} (\sec \theta \tan \theta d\theta) \\&= \frac{1}{4} \int \sec \theta \tan^2 \theta d\theta\end{aligned}$$

Compute the following integral showing all necessary work clearly.

$$\int \frac{1}{(x^2 + 9)^{5/2}} dx$$

$$\int \frac{dx}{\left[9\left(\left[\frac{x}{3}\right]^2 + 1\right)\right]^{5/2}}$$

$$\frac{x}{3} = \tan \theta$$

$$\frac{1}{3} dx = \sec^2 \theta d\theta$$

$$= \int \frac{3 \sec^2 \theta d\theta}{[3^2(\tan^2 \theta + 1)]^{5/2}}$$

$$= \int \frac{3 \sec^2 \theta d\theta}{[3^2 \sec^2 \theta]^{5/2}}$$

$$= \int \frac{3 \sec^2 \theta d\theta}{3^5 \sec^5 \theta}$$

$$= \frac{1}{3^4} \int \frac{1}{\sec^3 \theta} d\theta$$

$$= \frac{1}{3^4} \int \cos^3 \theta d\theta$$

$$= \frac{1}{3^4} \int \cos^2 \theta \cos \theta d\theta$$

$$= \frac{1}{3^4} \int (1 - \sin^2 \theta) \cos \theta d\theta$$

$$\text{Let } u = \sin \theta$$

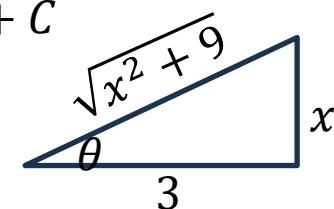
$$\Rightarrow du = \cos \theta d\theta$$

$$= \frac{1}{3^4} \int (1 - u^2) du$$

$$= \frac{1}{3^4} \left( u - \frac{1}{3} u^3 \right) + C$$

$$= \frac{1}{3^4} \left( \sin \theta - \frac{1}{3} \sin^3 \theta \right) + C$$

$$= \frac{1}{3^4} \left( \frac{x}{\sqrt{x^2+9}} - \frac{1}{3} \left[ \frac{x}{\sqrt{x^2+9}} \right]^3 \right) + C$$



# Examples

After an appropriate substitution, the integral

$\int x^2 \sqrt{9 - x^2} dx$  is equivalent to which of the following?

- (a)  $9 \int \cos^2 \theta d\theta$
- (b)  $81 \int \sin^2 \theta \cos^2 \theta d\theta$
- (c)  $27 \int \sin^2 \theta \cos \theta d\theta$
- (d)  $81 \int \sec^3 \theta \tan^2 \theta d\theta$
- (e)  $27 \int \sec^2 \theta \tan \theta d\theta$

Compute  $\int_0^4 \frac{x+2}{x^2+4} dx$ .

- (a)  $\frac{1}{2}(\ln 20 - \ln 4) + \arctan(2)$
- (b)  $\ln 6 - \ln 2$
- (c)  $\ln 20 - \ln 4$
- (d)  $\frac{1}{2}(\ln 20 - \ln 4) + 2 \arctan(4)$
- (e)  $\ln 20 - \ln 4 + 2 \arctan(4)$

Find  $\int \frac{x^2}{\sqrt{4 - x^2}} dx$

Which of the following is an appropriate substitution to use when solving the integral  $\int \sqrt{16x^2 - 9} dx$ ?

- (a)  $x = \frac{3}{4} \sin \theta$
- (b)  $x = \frac{4}{3} \sec \theta$
- (c)  $x = \frac{4}{3} \sin \theta$
- (d)  $x = \frac{3}{4} \sec \theta$
- (e)  $x = \frac{3}{4} \tan \theta$

Which of these substitutions would be used to evaluate  $\int x^2 \sqrt{x^2 + 4x + 13} dx$ ?

- (a)  $x + 4 = \sqrt{13} \sec \theta$
- (b)  $x + 2 = 3 \tan \theta$
- (c)  $x^2 + 4x = \sqrt{13} \tan \theta$
- (d) none of these.
- (e)  $x + 2 = 3 \sec \theta$

Evaluate  $\int \frac{1}{x^2 \sqrt{x^2 + 4}} dx$ .

# Examples

Compute  $\int \frac{1}{x^4\sqrt{x^2-4}} dx$ . In your final answer, any trig or inverse trig expressions that can be rewritten algebraically must be.

After an appropriate trigonometric substitution,

$$\int_{2\sqrt{2}}^4 \frac{\sqrt{x^2-4}}{x} dx$$
 is equivalent to

After an appropriate trigonometric substitution,

$$\int \frac{dx}{\sqrt{x^2+8x+41}}$$
 is equivalent to which of the following?

- (a)  $\frac{1}{5} \int \cos(\theta) d\theta$
- (b)  $\int \sec(\theta) d\theta$
- (c)  $\int \sec^2(\theta) d\theta$
- (d)  $\int \tan(\theta) d\theta$
- (e)  $\frac{1}{5} \int \sin(\theta) d\theta$

$$\text{Evaluate } \int \frac{1+x}{1+x^2} dx.$$

- (a)  $\frac{1}{2} \ln(1+x^2) + C$
- (b)  $\frac{3}{2} \ln(1+x^2) + C$
- (c)  $\ln(1+x^2) + C$
- (d)  $\frac{1}{2} \ln(1+x^2) + \arctan x + C$
- (e)  $\arctan x + \arcsin(x^2) + C$

$$(a) 2 \int_{\pi/4}^{\pi/3} \tan^2 \theta d\theta$$

$$(b) \int_{\pi/4}^{\pi/3} \sin(\theta) d\theta$$

$$(c) 2 \int_{\pi/4}^{\pi/6} \tan^2 \theta d\theta$$

$$(d) \int_{\pi/4}^{\pi/6} \sin(\theta) d\theta$$

(e) None of the above

Which of the following integrals is equivalent to  $\int \sqrt{4x^2 - 9} dx$ ?

- (a)  $2 \int \sec \theta \tan^2 \theta d\theta$
- (b)  $\frac{9}{2} \int \tan \theta d\theta$
- (c)  $\frac{9}{2} \int \sec \theta \tan^2 \theta d\theta$
- (d)  $\frac{9}{2} \int \sec^2 \theta \tan \theta d\theta$
- (e)  $2 \int \sec^2 \theta \tan \theta d\theta$

# Examples

After an appropriate substitution, the integral

$\int \sqrt{9-x^2} dx$  is equivalent to which of the following?

- (a)  $9 \int \sec \theta \tan^2 \theta d\theta$
- (b)  $3 \int \cos \theta d\theta$
- (c)  $9 \int \sec^3 \theta d\theta$
- (d)  $9 \int \cos^2 \theta d\theta$
- (e)  $3 \int \tan \theta d\theta$

If we use the appropriate trigonometric substitution to evaluate

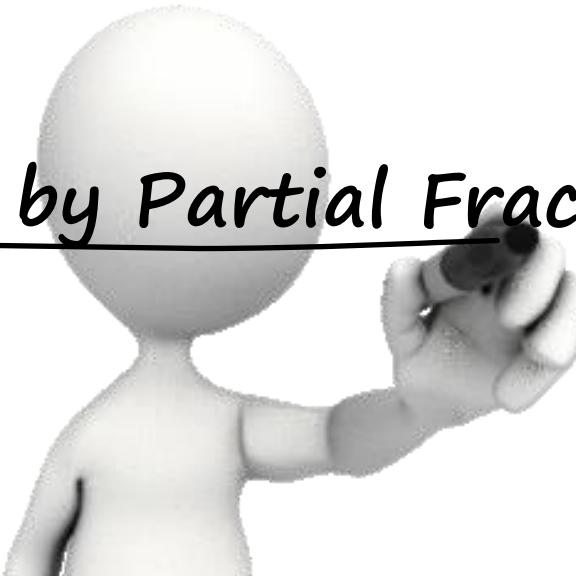
$\int_1^{2/\sqrt{3}} \left( \frac{\sqrt{x^2 - 1}}{x} \right) dx$ , which of the following is the correct result?

- (a)  $\int_0^{\pi/6} \tan^2 \theta d\theta$
- (b)  $\int_0^{\pi/6} \frac{\tan \theta}{\sec \theta} d\theta$
- (c)  $\int_0^{\pi/3} \tan^2 \theta d\theta$
- (d)  $\int_{\pi/2}^{\pi/6} \sin^2 \theta d\theta$
- (e)  $\int_{\pi/2}^{\pi/3} \sin^2 \theta d\theta$

Which of the following integrals is

equivalent to  $\int \frac{1}{(x^2 - 4x + 5)^{3/2}} dx$ ?

- (a)  $\frac{1}{9} \int \cos \theta d\theta$
- (b)  $\int \cos^3 \theta d\theta$
- (c)  $\frac{1}{27} \int \cos^3 \theta d\theta$
- (d)  $\int \sec \theta d\theta$
- (e)  $\int \cos \theta d\theta$



## Integration by Partial Fractions

Which of the following is the form of the partial-fraction decomposition for the rational function?

$$\frac{1}{(x+1)(x^2-2x-3)(x^2-2x+2)}$$

- (a)  $\frac{A}{x+1} + \frac{Bx+C}{x^2-2x-3} + \frac{Dx+E}{x^2-2x+2}$
- (b)  $\frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{D}{x-3} + \frac{Ex+F}{x^2-2x+2}$
- (c)  $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{D}{x^2-2x+2}$
- (d)  $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2-2x+2}$  ← correct
- (e)  $\frac{A}{x+1} + \frac{B}{x^2-2x-3} + \frac{C}{x^2-2x+2}$

$$\begin{aligned}& \frac{1}{(x+1)(x-3)(x+1)(x^2-2x+2)} \\&= \frac{1}{(x+1)^2(x-3)(x^2-2x+2)} \\&= \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2-2x+2}\end{aligned}$$

Compute the following integral showing all necessary work clearly.

$$\int \frac{4x^2 - 5x + 11}{(x+1)(x-1)(x^2+4)} dx$$

$$\begin{aligned} \frac{4x^2 - 5x + 11}{(x+1)(x-1)(x^2+4)} &= \frac{\frac{4+5+11}{(-2)(1+4)}}{(x+1)} + \frac{\frac{4-5+11}{(2)(1+4)}}{(x-1)} + \frac{Cx+D}{x^2+4} \\ &= \frac{\frac{20}{-10}}{(x+1)} + \frac{\frac{10}{10}}{(x-1)} + \frac{Cx+D}{x^2+4} \\ &= \frac{-2}{(x+1)} + \frac{1}{(x-1)} + \frac{Cx+D}{x^2+4} \end{aligned}$$

$$\frac{4x^2 - 5x + 11}{(x+1)(x-1)} = \left[ \frac{-2}{(x+1)} + \frac{1}{(x-1)} \right] (x^2 + 4) + Cx + D$$

$$\text{Let } x = 2i$$

$$\frac{-16 - 10i + 11}{5} = 2Ci + D$$

$$-1 - 2i = 2Ci + D$$

$$C = -1, D = -1$$

$$\int \left( \frac{2}{(x+1)} + \frac{1}{(x-1)} - \frac{x}{x^2+4} - \frac{1}{x^2+4} \right) dx$$

- $\int \frac{2}{(x+1)} dx = 2 \ln|x+1| + C$

- $\int \frac{1}{(x-1)} dx = \ln|x-1| + C$

- $\int \frac{x}{x^2+4} dx = \frac{1}{2} \ln(x^2+4) + C$

- $u = x^2 + 4 \Rightarrow du = 2x dx$

- $\int \frac{x}{x^2+4} dx = \frac{1}{2} \int \frac{1}{u} du$

- $\int \frac{1}{x^2+4} dx = \frac{1}{4} \int \frac{1}{\left(\frac{x}{2}\right)^2+1} dx$

- $\tan \theta = \frac{x}{2}$

- $2 \sec^2 \theta d\theta = dx$

$$= \frac{1}{4} \int \frac{2 \sec^2 \theta}{(\tan \theta)^2 + 1} d\theta$$

$$= \frac{1}{2} \int \frac{\sec^2 \theta}{\sec^2 \theta} d\theta$$

$$= \frac{1}{2} \theta + C$$

$$= \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$$

$$\int \frac{4x^2 - 5x + 11}{(x+1)(x-1)(x^2+4)} dx =$$

$$2 \ln|x+1| + \ln|x-1|$$

$$- \frac{1}{2} \ln(x^2 + 4) + \arctan\left(\frac{x}{2}\right) + C$$

# Exercise

Which of the following is a proper Partial Fraction Decomposition for the rational function

$$\frac{5x+1}{(x+3)(x^2+4x+3)(x^2+4)}$$

- (a)  $\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2+4}$
- (b)  $\frac{A}{x+3} + \frac{Bx+C}{x^2+4x+3} + \frac{Dx+E}{x^2+4}$
- (c)  $\frac{A}{x+3} + \frac{Bx+C}{(x+3)^2} + \frac{D}{x+1} + \frac{Ex+F}{x^2+4}$
- (d)  $\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x+1} + \frac{D}{x+2} + \frac{D}{x-2}$
- (e) None of these.

$$\int \frac{x^3+x}{x-1} dx =$$

- (a)  $\frac{x^3}{3} + \frac{x^2}{2} + 2x - 2 \ln|x-1| + C$
- (b)  $\frac{x^3}{3} - \frac{x^2}{2} + 2x + 2 \ln|x-1| + C$
- (c)  $\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \ln|x-1| + C$
- (d)  $\frac{x^3}{3} + \frac{x^2}{2} - 2x + 2 \ln|x-1| + C$
- (e)  $\frac{x^3}{3} - \frac{x^2}{2} + 2x - 2 \ln|x-1| + C$

Which of the following is a proper Partial Fraction Decomposition for  $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)}$ ?

- (a)  $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{Ax+B}{x^2-16} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$
- (b)  $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{x+4} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$
- (c)  $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{Ax+B}{x^2-16} + \frac{Cx+D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$
- (d)  $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{x+4} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{E}{x^2+1}$
- (e)  $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$

# Exercise

Compute  $\int \frac{2x^2 + 5x - 5}{(x+1)(x+3)^2} dx$

Find  $\int \frac{x+2}{x^2(x^2+1)} dx$

Write out the form of the partial fraction decomposition of the function

$$f(x) = \frac{x^3 - 2x^2 - 5x + 4}{(x+2)^2(x^2-1)(x^2+5x+7)}$$

(a)  $\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} + \frac{D}{x+1} + \frac{Ex+F}{x^2+5x+7}$

(b)  $\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{x^2-1} + \frac{Ex+F}{x^2+5x+7}$

(c)  $\frac{A}{(x+2)^2} + \frac{B}{x-1} + \frac{C}{x+1} + \frac{Dx+E}{x^2+5x+7}$

(d)  $\frac{A}{(x+2)^2} + \frac{B}{x^2-1} + \frac{Cx+D}{x^2+5x+7}$

(e)  $\frac{Ax+B}{(x+2)^2} + \frac{Cx+D}{x^2-1} + \frac{Ex+F}{x^2+5x+7}$

$$\int \frac{3-x}{x^2+3x-4} dx =$$

(a)  $\frac{-7}{5} \ln|x+4| + \frac{2}{5} \ln|x-1| + C$

(b)  $\frac{7}{2} \ln|x+4| + \frac{2}{5} \ln|x-1| + C$

(c)  $\frac{1}{5} \ln|x-4| - \frac{4}{5} \ln|x+1| + C$

(d)  $\frac{-1}{5} \ln|x-4| + \frac{4}{5} \ln|x+1| + C$

(e)  $\frac{2}{5} \ln|x+4| - \frac{7}{5} \ln|x-1| + C$

Evaluate  $\int_0^1 \frac{4x^2 + 5}{2x + 1} dx$

- (a)  $2 \ln 3$
- (b)  $3 \ln 3$
- (c)  $4 \ln 3$
- (d)  $6 \ln 3$
- (e) None of these

Evaluate  $\int \frac{-2x+4}{(x^2+1)(x-1)} dx$

Find  $\int \frac{2x^3 - 4x - 8}{(x^2 - x)(x^2 + 4)} dx$  showing all necessary work.