

# 2024 Fall Math 140 Week-In-Review

## Week 13: Final Exam Review

**Disclaimer:** This is by no means a comprehensive exam review. These problems do not cover all topics or all the ways in which the topics covered could be asked.

1. For the given matrices, perform the indicated operations, if possible.

$$A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} \quad B = \begin{bmatrix} -1 & x & 5 \\ y & 3 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 1 & z \end{bmatrix}$$

(a)  $3A - 2B^T$

(b)  $4CA$

2. Use the given matrix equation to solve for each variable.

$$5 \begin{bmatrix} (3-x) & 2 \\ 9y & -4 \end{bmatrix} + 6 \begin{bmatrix} -1 & 4 \\ 0 & 3z \end{bmatrix} = \begin{bmatrix} 11 & 34 \\ -8 & -7 \end{bmatrix}$$

3. The points  $(-4, -4)$  and  $(2, 7)$  form a line. Determine the equation of the line in point-slope form and standard form. Leave all numbers in exact form.

4. A particular item depreciates at a rate of \$118.27 per year. Two years after purchase, the item's value is \$1,488.46.

(a) Determine the purchase price of the item.

(b) If the scrap value of the item is \$665, how long will it take for the item to reach its scrap value? Round your answer to the nearest tenth of a year.

5. A business produces and sells Product Zeta. When the company produces 15 units of the product, the total cost is \$7,143.75 and when the company produces 25 units, the total cost is \$7,406.25. When the company sells 42 units of the product, the total revenue is \$3,507. Determine linear functions representing the total production cost,  $C(x)$ , and the total revenue,  $R(x)$ , for producing and selling  $x$  units of Product Zeta.

6. The cost and revenue functions for  $x$  units of particular product are given by  $C(x) = 122x + 11,117.25$  and  $R(x) = 213.5x$ , respectively. Determine the profit function,  $P(x)$ , for the product and discuss any break-even points.

7. For Item Z, the market price-demand function is given by  $D(x) = p(x) = -0.01x + 12.15$ . For the same item, producers are willing to supply 80 items at a price of \$4.35 each and are willing to supply 126 items at a price of \$5.50 each. Determine any equilibrium points for Item Z.

8. Determine the solution(s), if any, for the given system of equations. If the case of a parametric solution, use  $t$  as the parameter.

$$8x + 6y = -12$$

$$3y = -4x + \frac{3}{2}$$

9. Determine the solutions(s), if any, for the given system of equations. If the case of a parametric solution, use  $t$  as the parameter.

$$2x + 3z = 30 - 5y$$

$$0.6z - 6 = -0.4x - y$$

$$6x + z + 11 = 7y$$

10. Graph the given inequality.

$$3x + 5y > -10$$

11. For the given system of inequalities, determine the solutions set and its corner points, the point(s) where  $F = 3x + 6y$  is minimized, and the point(s) where  $G = 10x + 8y$  is maximized.

$$-80 \geq -5x - 4y$$

$$x \geq 28 - 2y$$

$$x \geq 0, y \geq 0$$

12. For the following standard maximization problem, rewrite all inequalities as equations using slack variables and set up the initial simplex tableau.

Maximize:  $H = 0.13x + 0.08y + 0.11z$

Subject To:

$$3x + 9y - 2z \leq 455$$

$$4x - 4y - 4z \geq -380$$

$$2x - y + 3z \leq 420$$

$$x \geq 0, y \geq 0, z \geq 0$$

13. For the given simplex tableau, determine any basic or non-basic variables, the corner point, the solution, and if it is the optimal solution. If it is not the optimal solution, determine the next pivot element.

$x$	$y$	$z$	$s_1$	$s_2$	$s_3$	$V$	
1	3	$-\frac{2}{3}$	$\frac{1}{3}$	0	0	0	$\frac{455}{3}$
0	16	$\frac{4}{3}$	$\frac{4}{3}$	1	0	0	$\frac{2960}{3}$
0	-7	$\frac{13}{3}$	$-\frac{2}{3}$	0	1	0	$\frac{350}{3}$
0	$\frac{31}{100}$	$-\frac{59}{300}$	$\frac{13}{300}$	0	0	1	$\frac{1183}{60}$



14. An experiment consists of drawing a marble out of a bag and recording the color (B=blue, G=green, and Y=yellow), then flipping a coin and recording the results (H=heads, T=tails).

(a) Write the sample space for the experiment. What are the total number of events for this experiment?

(b) If there 3 blue marbles, 2 green marbles, and 6 yellow marbles, what is the probability that the marble drawn is not yellow?

(c) Using the symbols above, write the symbolic notation of the event: "neither a blue or yellow marble is drawn, but the coin lands on tails".

15. Let  $J$  and  $K$  be two events of a sample space  $S$ . If  $P(J \cap K^C) = 0.24$  and  $P(J^C) = 0.48$ , determine the following:

(a)  $P(J \cap K)$

(b)  $P(K)$

(c)  $P(J^C \cap K^C)$

16. For the sample space  $S = \{x_1, x_2, x_3, x_4, x_5\}$ , it is known that  $P(x_1) = \frac{3}{17}$ ,  $P(x_2) = \frac{2}{17}$ ,  $P(x_4) = \frac{7}{17}$ , and  $P(x_5) = \frac{1}{17}$ .

(a) Construct a probability distribution for  $S$ .

(b) For the events  $A = \{x_1, x_4\}$  and  $B = \{x_1, x_3, x_5\}$ , determine  $P(A^C \cap B)$

(c) What is the expected value for the sample space?

17. A game consists of rolling a standard fair 4-sided die and then a standard fair 5-sided die. The game costs \$4 to play. If you roll a sum of 9, you win \$9. If you roll a double, you win \$8. If you roll a sum of 5 you win \$5. With any other roll you don't win.

(a) Construct a probability distribution for the game where  $X$  is the net winnings.

(b) What is the expected net winnings for the player in this game? Is this a fair game?

18. For the given polynomial functions, determine the domain, degree, leading coefficient, constant term, and end-behavior.

(a)  $f(x) = -10x^3 + 7x^4 - 2x^9 + e^{13}$

(b)  $g(x) = 3x(x - 9)(x - 5)^2(x + 2)(x + 4)$

19. For the given functions, determine the domain, the vertex, the minimum, the maximum, the y-intercept, and any real roots/zeros.

(a)  $f(x) = 5x^2 + 8x - 3$

(b)  $g(x) = (7 - 2x)(x - 4)$

20. For the given functions, determine the domain, any intercepts, any holes, and any vertical asymptotes.

$$(a) \ h(x) = \frac{x^2 - 25}{2x^2 + 7x - 15}$$

$$(b) \ k(x) = \frac{2x(x - 3)(x + 7)^2}{-9x(x - 3)^2(x + 1)}$$

21. Fully simplify the given expression.

$$2 \left( \frac{x^2 - 9}{x + 1} \right) \left( \frac{x}{x + 3} \right) - \frac{x(2x + 1)}{x - 2} \div \frac{x + 1}{x - 2}$$

22. Fully simplify the given expression. Write your answer using only positive exponents.

$$\left(\frac{28x^{-4}y^5}{(2xy)^3}\right)^{-3/2}$$

23. Rewrite the following functions in radical form and determine their domain.

(a)  $f(x) = 2(19 - 4x)^{7/6}$

(b)  $g(x) = \frac{5x}{(7x + 2)^{-4/3}}$

24. Setup and begin to simplify the difference quotient for the given functions.

(a)  $f(x) = 2x^2 - 5x + 7$

(b)  $g(x) = \frac{9}{1-x}$

(c)  $h(x) = \sqrt{8x+3}$



25. Rewrite  $j(x) = 7|8 - 3x| + 2$  as a piecewise function.

26. Fully simplify the given expression.

$$\frac{125^x \cdot 2^{3x-11}}{5^{2-x} \cdot 16^{x-2}}$$

27. Algebraically solve the given equation.

$$27^{2x-1} = \frac{81}{3^{-5x}}$$

28. Write the function  $h(x)$  that is the parent function  $f(x) = \sqrt[3]{x}$  with the following transformations:

- shift left 5 units
- vertical compression by a factor of  $\frac{5}{3}$
- shift down 4 units

29. Use the given functions to determine the following.

$$f(x) = 2\sqrt{4x + 1} \qquad g(x) = \frac{x + 3}{x - 1}$$

(a)  $(f + g)(2)$

(b)  $\left(\frac{f}{g}\right)(0)$

(c)  $f(g(-1))$

30. Write the given expression as a single logarithmic term.

$$4 + 9 \log_3(x) - 2 \log_3(x + 7) + \log_3(x - 5)$$

31. Algebraically solve the given equation.

$$\log_3(x^2 - 3) = 1 + \log_3(x - 1)$$

32. For the given functions, determine the domain.

$$(a) f(x) = \sqrt[3]{5x+2} - \frac{x+2}{x-1} + \ln(x+11)$$

$$(b) g(x) = \frac{5\sqrt[4]{-5x+28}}{3\log_6(2x+9)}$$

33. How long will it take \$5,000 to grow to \$12,000 as a one-time investment at an annual interest rate of 4.3% compounded continuously?

34. You borrow \$2,500 as a short-term simple interest loan for 25% interest to be paid back in 30 months. How much will you have to pay back in 30 months?

35. From the following accounts, which would be the best for a loan?

- Account A: 3.8% annual interest, compounded monthly
- Account B: 3.72% annual interest, compounded weekly
- Account A: 3.85% annual interest, compounded continuously

36. How much would you need to deposit in a savings account if the account has a 5.2% annual interest rate compounded monthly and you want to have \$25,000 after 7 years?

37. To help with your eventual retirement, you open an account with an initial deposit of \$2,700 when you are 25 years old. You then make deposits of \$150 in the account each month until you are 50 years old. How much money is in the account at that point? How much did you earn in interest from the account?

38. Your business wants to purchase an office building that costs \$850,000. The bank is willing to finance your business for 82% of the cost at an annual interest rate of 3.3% compounded quarterly for 10 years. If your business takes out the maximum loan possible, what will be the monthly payment (to the nearest cent) your business will have to make to the bank each quarter? What will be the total paid for the building at the end of the loan?