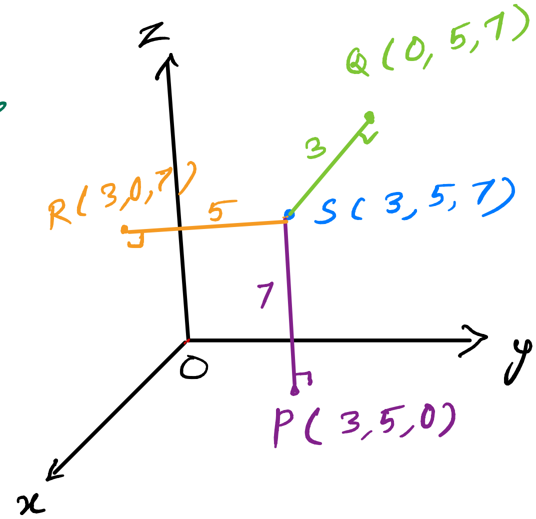
Example 1 (12.1)**.** *Let P, Q, and R be the projections of the point $S(3,5,7)$ onto xy-plane, yz-plane and xz-plane, respectively. Determine the coordinates of the points P, Q, and R, and compute the distance from the origin to the point S.*

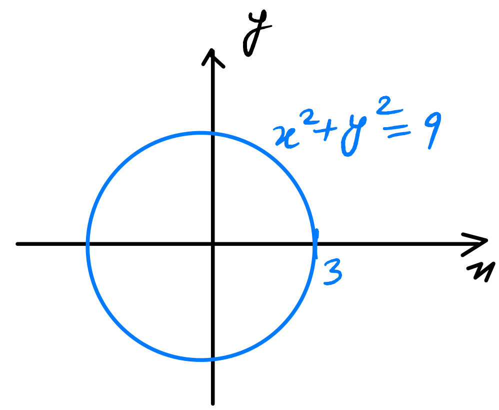
The distance from the origin to the point S is $|OS| = \sqrt{(3-0)^2+(5-0)^2+(7-0)^2}$

$= \sqrt{83}$ units.

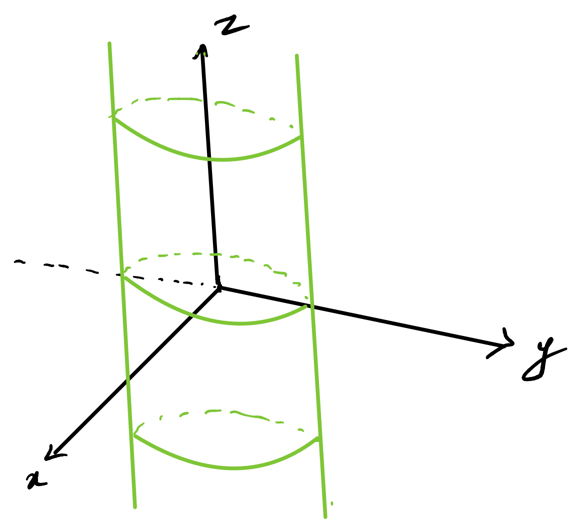


Example 2 (12.1)**.** *(a) Sketch the graph of $x^2 + y^2 = 9$ in $\mathbb{R}^2$.*
(b) Sketch the graph of $x^2 + y^2 = 9$ in $\mathbb{R}^3$.
(c) Sketch the graph of $y^2 + z^2 = 1$, $x \geq 2$ in $\mathbb{R}^3$.

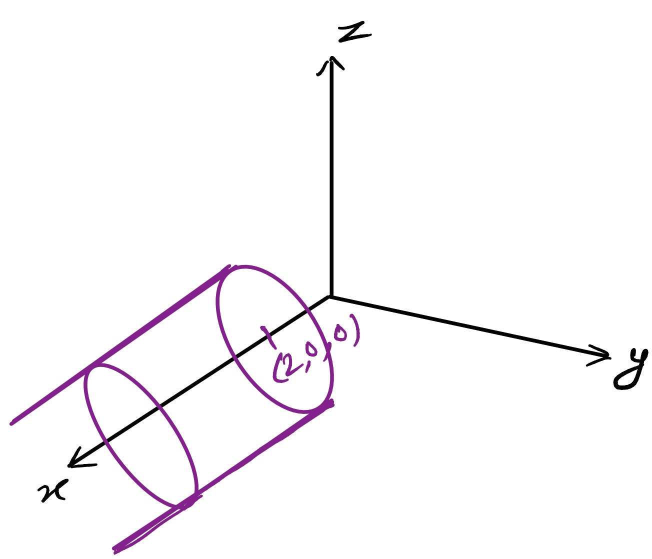
(a)



(b)



(c)



Example 3 (12.1). *Let the sphere S_1 is given by the equation $x^2 + y^2 + z^2 + 2x - 4z = 11$. Find the distance between the center of the sphere S_1 and the point $P(1, 4, 6)$.*

S_1: $x^2 + y^2 + z^2 + 2x - 4z = 11$

$x^2 + 2x + 1 + y^2 + z^2 - 4z + 4 = 11 + 1 + 4$

$(x+1)^2 + (y-0)^2 + (z-2)^2 = 16$

Center of the sphere is $C(-1, 0, 2)$.

$|CP| = \sqrt{(-1-1)^2 + (0-4)^2 + (2-6)^2} = 6$ units.

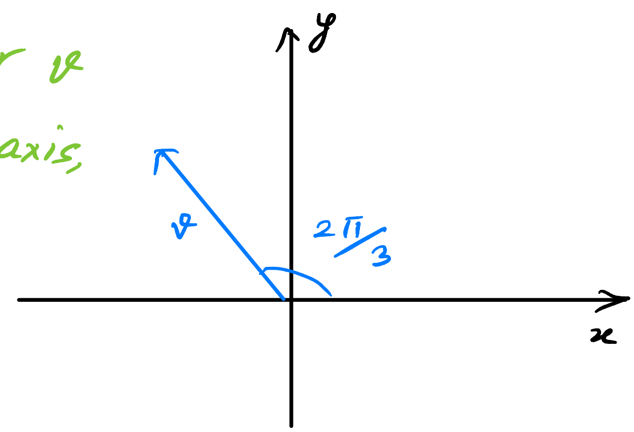
Example 4 (12.2). *The initial point of a vector* $\mathbf{v}$ *in* $\mathbb{R}^2$ *is the origin and the terminal point is in the quad II. If* $\mathbf{v}$ *makes an angle of* $\frac{2\pi}{3}$ *with positive* x*-axis and* $|\mathbf{v}| = 6$*, find the vector* $\mathbf{v}$.

As a matter of fact, if a vector v makes angle θ with positive x-axis, then we can write v as

$v = \langle |v| \cos\theta, |v| \sin\theta \rangle$

or

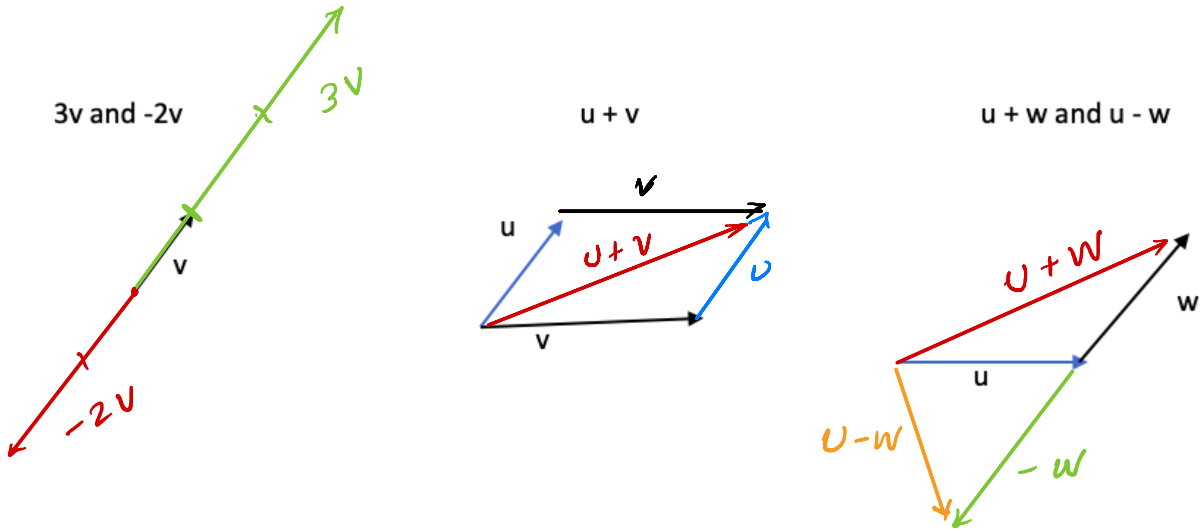
$= |v| (\cos\theta\, i + \sin\theta\, j)$.



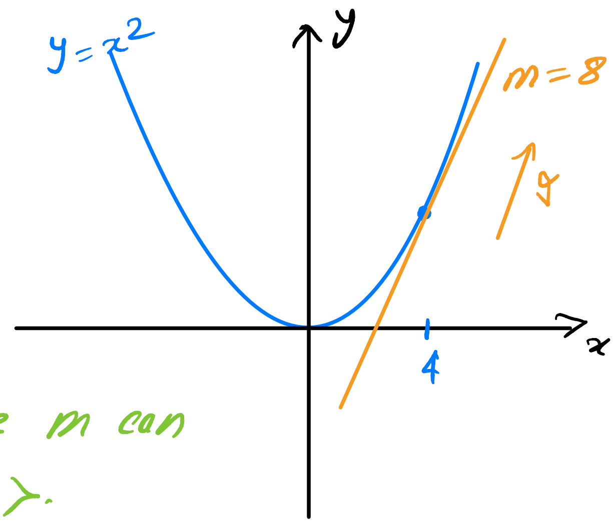
so, $v = 6\left(\cos\frac{2\pi}{3} i + \sin\frac{2\pi}{3} j\right) = 6\left(-\frac{1}{2} i + \frac{\sqrt{3}}{2} j\right)$

$= -3i + 3\sqrt{3}j$

Example 5 (12.2)**.** *Draw the vectors as described below.*



Example 6 (12.2)**.** *Find the unit vectors that are parallel to the tangent line to the parabola* $y = x^2$ *at the point* $(4, 16)$.



The slope of the tangent line to the curve $y = x^2$ at $(4, 16)$ is

$$m = \left.\frac{dy}{dx}\right|_{x=4} = 2x\Big|_{x=4} = 8$$

A vector parallel to the line with slope m can be written as $1i + mj = \langle 1, m \rangle$.

So, $v = \langle 1, 8 \rangle$ is a vector parallel to the tangent line, and $u = \frac{v}{|v|} = \frac{1}{\sqrt{65}}\langle 1, 8 \rangle$ is a unit vector parallel to the tangent.

So, $\pm \frac{1}{\sqrt{65}}\langle 1, 8 \rangle$ are the desired unit vectors.

Example 7 (12.3). *If* $\mathbf{a} = < \mathbf{2}, \mathbf{-1}, \mathbf{0} >$, *find a vector* $\mathbf{b}$ *such that* $comp_{\mathbf{a}}\mathbf{b} = 3$.

Suppose $b = \langle x, y, z \rangle$.

$$Comp_a b = \frac{a \cdot b}{|a|} \Rightarrow 3 = \frac{2x - y + 0}{\sqrt{5}}$$

$$\Rightarrow y = 2x - 3\sqrt{5}$$

Consider $x = t$ and $z = s$, where $s, t \in \mathbb{R}$.

This means s and t are real numbers.

Then, $b = \langle t, 2t - 3\sqrt{5}, s \rangle$, where $s, t \in \mathbb{R}$.

In particular, choosing $s = t = 0$, $b = \langle 0, -3\sqrt{5}, 0 \rangle$.

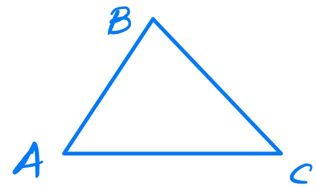
Example 8 (12.3). *Find the direction angles of the vector* $\mathbf{a} = < \overset{a_1}{1}, \overset{a_2}{2}, \overset{a_3}{-1} >$.

$$\cos\alpha = \frac{a_1}{|a|} = \frac{1}{\sqrt{6}} \Rightarrow \alpha = \cos^{-1}\left(\frac{1}{\sqrt{6}}\right).$$

$$\cos\beta = \frac{a_2}{|a|} = \frac{2}{\sqrt{6}} \Rightarrow \beta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$$

$$\cos\gamma = \frac{a_3}{|a|} = \frac{-1}{\sqrt{6}} \Rightarrow \gamma = \cos^{-1}\left(\frac{-1}{\sqrt{6}}\right).$$

Example 9 (12.3). *Use vectors to determine whether the triangle with vertices $A(3, 2, 0)$, $B(0, 1, 2)$ and $C(3, 1, 2)$ is right-angled.*



$\overrightarrow{AB} = \langle 0-3, 1-2, 2-0 \rangle = \langle -3, -1, 2 \rangle$

$\overrightarrow{AC} = \langle 0, -1, 2 \rangle$

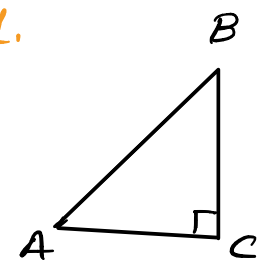
$\overrightarrow{BC} = \langle 3, 0, 0 \rangle$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 0 + 1 + 4 = 5$

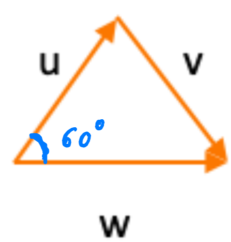
$\overrightarrow{AB} \cdot \overrightarrow{BC} = -9$

$\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$; so the vectors are orthogonal.

Hence, the triangle ABC is right-angled.



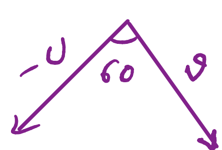
Example 10 (12.3). *Consider the triangle below is an equilateral triangle with* $|\mathbf{u}| = 1$. *Compute* $\mathbf{u} \cdot \mathbf{v}$ *and* $\mathbf{u} \cdot \mathbf{w}$.



Note that $|u| = |v| = |w| = 1$.

$u \cdot w = |u||w| \cos 60^\circ = (1)(1)\left(\frac{1}{2}\right) = \frac{1}{2}$

$$\boxed{u \cdot w = \frac{1}{2}}$$



$-u \cdot v = |-u||v| \cos 60^\circ$

$-u \cdot v = (1)(1)\left(\frac{1}{2}\right)$

$$\boxed{u \cdot v = -\frac{1}{2}}$$