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SECTION 3.1: SETTING LINEAR PROGRAMMING PROBLEMS

- Always Define Your Variables → look at the final question in the problem..
- Objective Function
- Constraints

Pr 1. Set up, but do not solve. → 3 variables

A housing contractor wants to develop a 60 acre tract of land. He has three types of houses: a two-bedroom, a three-bedroom and a four-bedroom house. The two-bedroom house requires $70,000 of capital for a profit of $20,000, the three-bedroom house requires $84,000 of capital for a profit of $25,000, and the four-bedroom house requires $100,000 of capital for a profit of $24,000. The two-bedroom needs 3000 labor hours, the three-bedroom needs 3500 labor hours, and the four-bedroom house needs 3900 labor hours. There are currently 250,000 labor hours available. If the two-bedroom house is on half an acre, the ~~large four~~ 3-bedroom house is on 0.75 acres, the four-bedroom house is on 1.5 acres and the contractor has 6 million in capital, how many of each type of house should be built to maximize the profit?

→ profit

variables ⇔ each type of house

Variables:

x := the number of 2-bedroom houses

y := the number of 3-bedroom houses

z := the number of 4-bedroom houses

P := the total profit

⋮

Objective: Maximize/ Minimize $P = 20000x + 25000y + 24000z$

Subject to:

$.5x + .75y + 1.5z \le 60$ (total acres)

$70000x + 84000y + 100000z \le 6000000$ (total capital)

$3000x + 3500y + 3900y \le 250000$ (total labor hours)

$x \ge 0,\ y \ge 0,\ z \ge 0$ (non-negativity constraints)

Pr 2. Set up, but do not solve.

Your burger company sells three different types of patty melts - the Big cheesy, the double decker, and the classic. These patty melts all use different amounts of cheese (slices), bread (slices), and patties, as given in the table.

is a constraint

	Cheese	Bread	Patties
Big Cheesy	3	2	2
Double Decker	2	3	2
Classic	1	2	1

} Table

rows is a variable

The profit for the Big Cheesy is $1, for the Double Decker is $2 and for the Classic is $1. Due to certain agreements, the company can make at most 250 Double Deckers. If the company has 300 slices of cheese, 600 slices of bread, and 800 beef patties, how many of each type of patty melt should be produced in order to maximize the profit?

- profit is a variable
- variable for each type of patty melt.

"at most" $\leq$

"at least" $\geq$

"more than" $>$

P = the total profit

B = the number of Big cheesies produced

D = the number of Double Deckers Produced

C = the number of classics produced

Maximize $P = 1B + 2D + 1C$ $\quad (P = B + 2D + C)$

Subject to:

$3B + 2D + 1C \leq 300$ (total cheese)

$2B + 3D + 2C \leq 600$ (total bread)

$2B + 2D + 1C \leq 800$ (total patties)

} optional

$D \leq 250$ (licensing agreements)

$B \geq 0,\ D \geq 0,\ C \geq 0$

Pr 3. Set up, but do not solve.

You have \$16,000 to invest, some in Stock A, some in Stock B, and some in Stock C. You have decided that the money invested in Stock A must be at least twice as much as that in Stock C. However, the money invested in Stock A must not be greater than \$9,000. If Stock A earns 3% annual interest, Stock B earns 6% annual interest, and Stock C earns 4 % annual interest, how much money should you invest in each stock to maximize your annual interest?

Need one variable per stock (money invested)

A = total amount invested in stock A
B = total amount invested in stock B
C = total amount invested in stock C
I = total annual interest, (in dollars)

Maximize $I = .03A + .06B + .04C$ $(3\% = .03)$

Subject to

$1 \cdot A + 1 \cdot B + 1 \cdot C \leq 16000$ (total to invest)

$A \geq 2C$ (ratio)

$A - 2C \geq 0$
$-A + 2C \leq 0$
same

$A \leq 9000$ (stock A)

$A \geq 0, \ B \geq 0, \ C \geq 0$

Suppose $C = 1, \ A = 2$
$2 \geq 2 \cdot 1$ ✓
$\rightarrow A \geq 2C$
at least $A \leq 2C$
$\rightarrow 2A \geq C$
$2A \leq C$
$2 \cdot 2 \geq 1$ (false)

Pr 4. Set up, but do not solve.

An independent soda company makes two soda flavors: big maroon and Gig'em Ginger. Each can of soda requires 2 cups of carbonated water. The Big Maroon uses three tablespoons of sugar, while Gig'em Ginger uses one tablespoon of sugar. Due to limitations on flavor packets, they can only produce 70 cans of Big Maroon. Suppose that they have 240 cups of carbonated water, and 160 tablespoons of sugar. If they sell each can of Big Maroon for $1 and each can of Gig'em Ginger for $0.40, how much of each type of soda should they make in order to maximize profit? Will they have any leftovers?

M = the number of cans of Big Maroon

G = the number of cans of Gig'em Ginger

P = the total profit

Maximize $P = 1M + .4G$

subject to:

$2M + 2G \leq 240$ (carbonated water)

$3M + 1G \leq 160$ (sugar)

$M \leq 70$ (flavor packets)

$M \geq 0, \; G \geq 0$

Section 3.2: Graphing Systems of Linear Inequalities in Two Variables

- Solution set to a linear inequality is half of the coordinate plane, while the solution set to a system of linear inequalities is the region of points that satisfy **all** of the linear inequalities in the system.
- Boundary Line - the corresponding linear equation for a a linear inequality
- True Shading vs. Reverse Shading
- Unbounded vs. Bounded solution sets
- Corner Points

Pr 1. Graph the inequality $4x - 9y < 24$, labeling the boundary line and the solution set with **S**.

dashed line

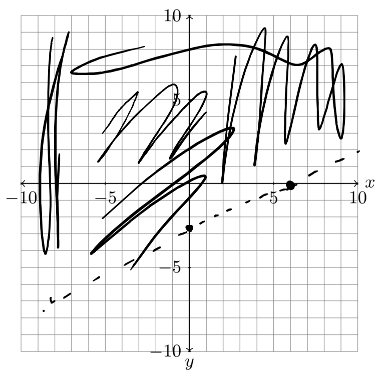
graph boundary line

$4x - 9y = 24$

($\leq, \geq$ - solid line)

($<, >$ - dashed line)

True shading



$4x - 9y = 24$?

(y-intercept) $x = 0$ → $4 \cdot 0 - 9 \cdot y = 24$

$\frac{-9y}{-9} = \frac{24}{-9}$

$y = -\frac{24}{9} = -\frac{8}{3}$ $(0, -8/3)$

$x=0$ $y=0$

$4 \cdot 0 - 9 \cdot 0 = 0 < 24$ ✓

x-intercept → $y=0$ $\frac{4x}{4} = \frac{24}{4}$ → $(6, 0)$

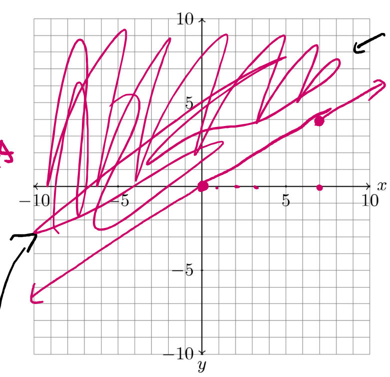
Pr 2. Graph the inequality $-4x + 7y \geq 0$, labeling the boundary line and the solution set with **S**.

solid line

check (a, b) not on line

if inequality is true, shade that half.

reverse shading



boundary line:

$-4x + 7y = 0$

$x = 0$ → $-4 \cdot 0 + 7y = 0$

$7y = 0$

$y = 0$ $(0, 0)$

"Need a second" point?

$-4x + 7y = 0$
$+4x$ $\quad$ $+4x$

→ $\frac{7y}{7} = \frac{4x}{7}$

$y = \frac{4}{7}x$ → slope

Pick a test point (not on the line)

pick $(1, 0)$

$-4 \cdot 1 + 7 \cdot 0 \geq 0$

$-4 \geq 0$ (No...)

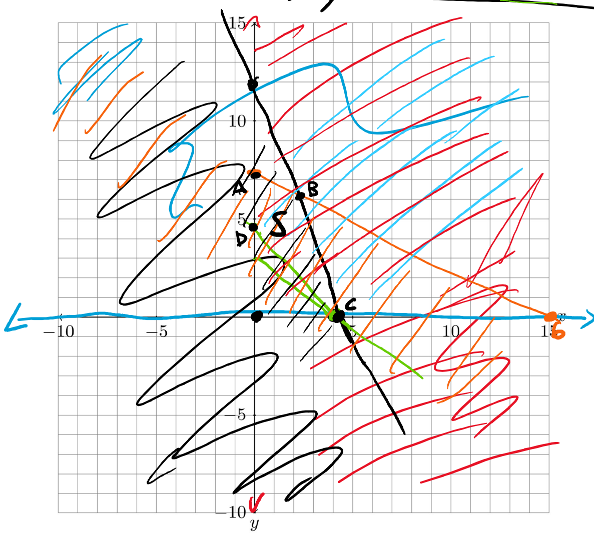
not in solutions

Pr 3. Graph the system of linear inequalities, the solution set with **S**. Then state whether the solution set is bounded or unbounded and the exact corner point(s) of the solution set.

$$3x + y \le 12$$
$$6x + 5y \ge 24$$
$$x + 2y \le 16$$
$$x \ge 0, y \ge 0$$

	$3x + y \le 12$	$6x + 5y \ge 24$	$x + 2y \le \cancel{15}$ 16	$x \ge 0$	$y \ge 0$
Boundary Line:	$3x+y=12$	$6x+5y=24$	$x+2y=16$	Skip	
***x*-intercept:** $y=0$	$3x=12 \to x=4$ $(4,0)$	$6x=24$ $x=4$ $(4,0)$	$x=16$ $(16,0)$		
***y*-intercept:** $x=0$	$y=12$ $(0,12)$	$5y=24$ $y=\frac{24}{5}=4+\frac{4}{5}$ $(0, 4.8)$	$2y=16$ $y=\frac{15}{2}=\cancel{7.5}$ 8 $(0,\cancel{7.5}$ 8$)$		
Test Point: $(0,0)$	$0 \le 12$ ✓	$0 \ge 24$ ✗	$0 \le 16$		

left-hand size becomes 0



Steps:
True shading
plug in test point into inequality
if the inequality is true
Shade the side the point is in.
(otherwise shade opp.)

Corner Points: Needed for section 3.3

3x

$A = (0, 8)$
$B = \left(\frac{8}{5}, \frac{36}{5}\right)$
$C = (4, 0)$
$D = (0, 4.8)$

$B \to$ $x + 2y = 16$
$3x + y = 12 \to y = 12 - 3x$

$x + 2(12 - 3x) = 16$
$x + 24 - 6x = 16$
$-5x = -8$
$x = 8/5$

$y = 12 - 3 \times \frac{8}{5} = \frac{36}{5}$

Pr 4. Use the graph below to write the corresponding system of linear inequalities.

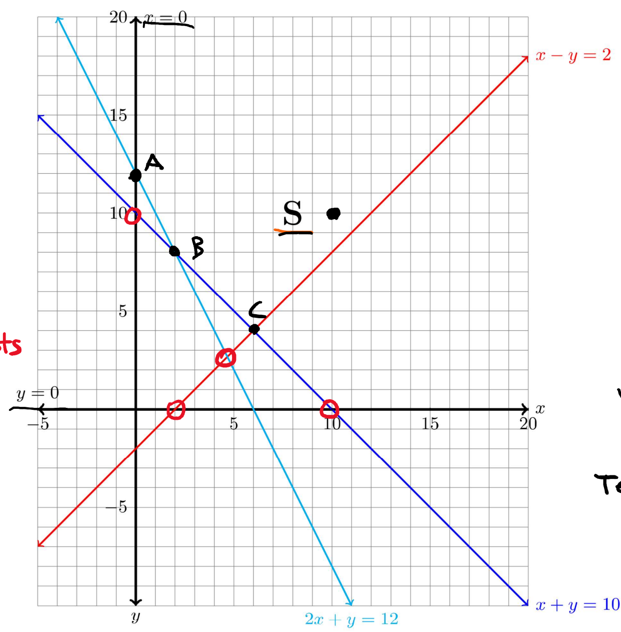
$x - y \leq 2$

$x + y \geq 10$

$2x + y \geq 12$

$x \geq 0, \quad y \geq 0$ ✓

O - not corner points



use a test point in S

for each line, we want the inequality a test point to be true

Test point (10, 10)

$x+y \overset{?}{} 2$

$10 - 10 = \cancel{20}\ 0 \ \overset{?}{\leq}\ 2$

$20 \geq 2$

$x+y \overset{?}{} 10$

$10+10 = 20 \geq 10$

$2(10) + 10 \geq 12$

30

. Find corner points

A is the y-intercept for $2x+y=12$

$x=0 \rightarrow y = 12$

$A = (0, 12)$ ✓

$B \overset{?}{=} (2,8)$ intersection

$\left.\begin{array}{l} 2x+y=12 \\ x+y=10 \end{array}\right\} \rightarrow$ use calc. to find RREF

$x = -y+10$ substitute into (1)

$2(-y+10) + y = 12$

$-2y + 20 + y = 12$

$-y = 12-20 = -8$

$y = 8$

$x = -8+10 = 2$

$(2,8)$ ✓

$C = (6,4)$

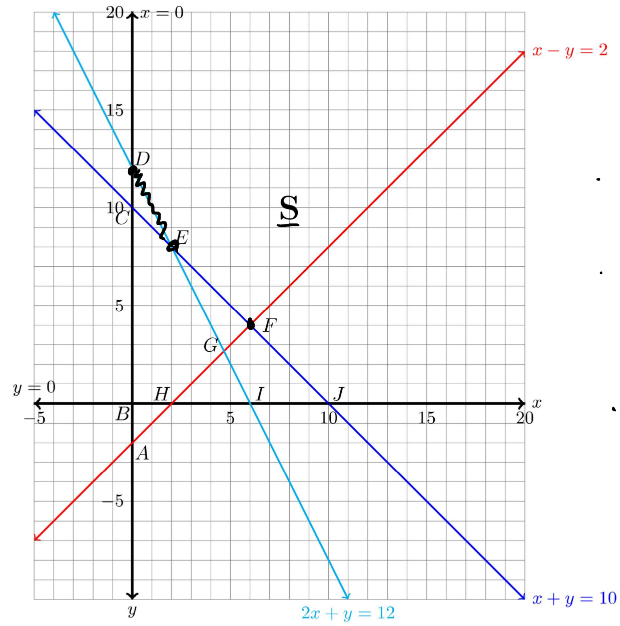
check $x - y = 2 \rightarrow 6-4 = 2$ ✓

$x + y = 10 \qquad 6+4 = 10$ ✓

Section 3.3: Graphical Solution of Linear Programming Problems

- Feasible Region
- Know the parts of the Fundamental Theorem of Linear Programming
- Method of Corners
  - Set up a linear programming problem algebraically. → Section 3.1
  - Graph the constraints and determine the feasible region. → Section 3.2
  - Identify the exact coordinates of all corner points of the feasible region.
  - Determine whether or not the linear programming problem will have a solution.
  - If a solution will exist, evaluate the objective function at each corner point and determine the optimal point.
- Leftovers

Pr 1. Use the feasible region to determine the maximum and minimum values of the objective function $z = 2x+y$ over the region, if they exist and where they occur.



Fund. Theorem

- if S is bounded, a maximum exists
- if S is unbounded * then there is no maximum
- a minimum always exists

the optimum happens at corner points ✓

S is unbounded → no max

$\boxed{\text{No Maximum}}$

(x, y)	Z = 2x+y	
A: $(0, -2)$	Not used	
B: $(0, 0)$		
C: $(0, 10)$		
D: $(0, 12)$	2(0) + (12)	= 12
E: $(2, 8)$	2(2) + (8)	= 12
F: $(6, 4)$	2(6) + 4	= 16
G: $(\frac{14}{3}, \frac{8}{3})$	Not used	
H: $(2, 0)$		
I: $(6, 0)$		
J: $(10, 0)$		

where is the min'm?

Minimum value of 12 occurs on the line segment connecting (0, 12) to (2, 8). } Answer 1

almost not allowed: infinitely many solutions

$2x + y = 12 \rightarrow y = -2x + 12, \quad x = t$

Answer 2: $(t, -2t+12)$, where $0 \le t \le 2$

Pr 2. Use the Method of Corners to solve the following linear programming problem. (need to graph)

Objective: Maximize $P = 12x + 4y$

Subject to: $3x + y \le 12$

$6x + 5y \ge 24$

$x + 2y \le 16$

$x \ge 0,\ y \ge 0$

graph from earlier

The region is bounded ✓

So a max exists.

		$P = 12x + 4y$
A	$(0, 8)$	$12(0) + 4(8) = 32$
B	$(1.6, 7.2)$	$12(1.6) + 4(7.2) = 48$
C	$(0, 4.8)$	$12 \cdot 0 + 4(4.8) = 19.2$
D	$(4, 0)$	$12 \cdot 4 + 4 \cdot 0 = 48$

$$\begin{bmatrix} 0 & 8 \\ 1.6 & 7.2 \\ 6 & 4.8 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 4 \end{bmatrix} =$$

Maximum value is 48, which occurs on the line segment from $(1.6, 7.2)$ to $(4, 0)$.

Pr 3. An independent soda company makes two soda flavors: big maroon and Gig'em Ginger. Each can of soda requires 2 cups of carbonated water. The Big Maroon uses three tablespoons of sugar, while Gig'em Ginger uses one tablespoon of sugar. Due to limitations on flavor packets, they can only produce 70 cans of Big Maroon. Suppose that they have 240 cups of carbonated water, and 160 tablespoons of sugar. If they sell each can of Big Maroon for \$1, and each can of Gig'em Ginger for \$0.40, how much of each type of soda should they make in order to maximize profit? Will they have any leftovers?

we already set up the problem

M = # of Maroon cans

G = # of Ginger cans

P = profit

Maximize $P = M + .4G$

subject to

$$2M + 2G \le 240$$
$$3M + G \le 160$$
$$M \le 70$$
$$M \ge 0, \quad G \ge 0$$

$2M + 2G = 240$

$M + G = 120$

$G = -M + 120$ *

$3M + G = 160$

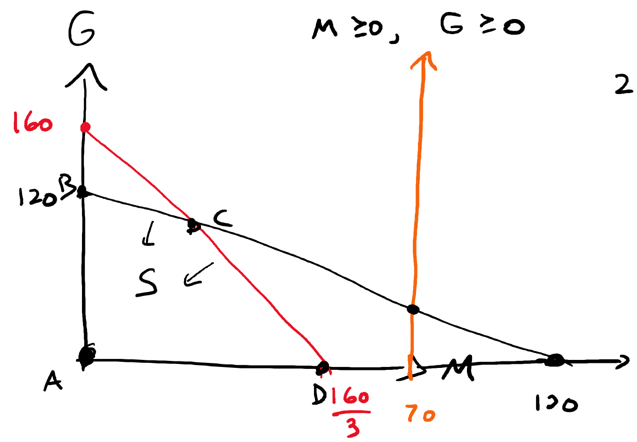
$G = -3M + 160$ <

$M = 70$

$0 = -3M + 160 = 0$

$-3M = -160$

$M = \frac{-160}{-3} = \frac{160}{3} \approx 53$ ish



Test point : (0,0)

$2 \cdot 0 + 2 \cdot 0 \le 240$ ✓

$3 \cdot 0 + 0 \le 160$

$0 \le 70$ ✓

S is bounded, so max exists

		$P = M + .4G$
A	(0,0)	0
B	(0, 120)	$0 + .4 \times 120 = 48$
C	(20,100)	$20 + .4 \times 100 = 60$
D	$(\frac{160}{3}, 0)$	$\frac{160}{3} + .4 \cdot 0 = 53.\overline{33}$

C → intersection

$$\begin{cases} 2M + 2G = 240 \\ 3M + G = 160 \end{cases}$$

(20,100)

use RREF

$\boxed{\text{Maximum profit is \$60 for selling 20 Big Maroon cans and 100 Gig'em ginger cans}}$

For left-overs, we plug in the solution into each constraint and then calculate the difference.

$2G + 2M \le 240$

$2(20) + 2(100) = \underline{240} = 240$ 0 leftover cups of carbonated water

$3M + G \leq 160$

$3(20) + 100 = 160$ → No leftover sugar.

$M \leq 70$

$20 \leq 70$

$70 - 20 = 50$

50 leftover flavor packets...