## Review of Sections 3.7, 3.8, 3.9

1. The ball is tossed into the air. Its position at time $t$ is given by $\mathbf{r}(t)=<5 t, 100 t-16 t^{2}>$.
(a) Find the velocity and the speed of the ball when $t=2$.
(b) How high does the ball go?
(c) With what speed does the ball hit the ground?
2. If $\mathbf{r}(t)=<t^{3}, t^{2}>$ represents the position of a particle at time $t$, find the angle between the velocity and the acceleration vector at time $t=1$.
3. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of $60 \mathrm{~cm} / \mathrm{s}$. Find the rate at which the area within the circle is increasing after 5 sec.
4. If a ball is thrown vertically upward with a velocity of $144 \mathrm{ft} / \mathrm{s}$, then its height after $t$ seconds is $s=$ $144 t-16 t^{2}$.
(a) What is the maximum height reached by the ball?
(b) What is the velocity of the ball when it is 320 ft above the ground on his way up?
(c) What is the velocity of the ball when it is 320 ft above the ground on his way down?
(d) When will the ball hit the ground?
(e) With what velocity does the ball hit the ground?
5. A bacteria culture starts with 1000 bacteria and the growth rate is proportional to the number of bacteria. After 2 h the population is 9000 .
(a) Find an expression for the number of bacteria after $t$ hours.
(b) Find the number of bacteria after 3 h .
(c) In what period of time does the number of bacteria double?
6. An isotope of strontium, $\mathrm{Sr}^{90}$, has a half-life of 25 years.
(a) Find the mass of $\mathrm{Sr}^{90}$ that remains from a sample of 18 mg after $t$ years.
(b) How long will it take for the mass to decay to 2 mg ?
7. A cup of coffee has a temperature of $200^{\circ} \mathrm{F}$ and is in a room that has a temperature of $70^{\circ} \mathrm{F}$. After 10 min the temperature of the coffee is $150^{\circ} \mathrm{F}$.
8. A balloon is rising at a constant speed of $5 \mathrm{ft} / \mathrm{s}$. A boy is cycling along a straight road at a speed of 15 $\mathrm{ft} / \mathrm{s}$. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?
9. A kite 100 ft above the ground moves horizontally at a speed of $8 \mathrm{ft} / \mathrm{s}$. At what rate is the angle between the spring and the horizontal decreasing when 200 ft of string have been let out?
10. A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across the top and have a height of 1 ft . If the trough is filled with water at a rate of $12 \mathrm{ft}^{3} / \mathrm{min}$, how fast is the water level rising when the water is 6 inches deep?
11. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If the water is poured into the cup at a rate of $2 \mathrm{~cm}^{3} / \mathrm{s}$, how fast is the water level rising when the water is 5 cm deep?

## Review for Exam 2.

1. An object is moving along a straight path. The position of the object at time $t$ is given by

$$
s(t)=2 t^{3}-9 t^{2}+12 t+1
$$

, where $t$ is measured in seconds and $s(t)$ is measured in feet. Find
(a) the velocity and acceleration as functions of $t$.
(b) the acceleration when the velocity is zero.
(c) the total distance traveled in the first 2 seconds.
2. At what point on the curve $f(x)=36 \sqrt{x}$ is the tangent line parallel to the line $9 x-y+2=0$ ?
3. Find an equation of the tangent line to the curve $2 e^{x y}=x+y$ at the point $(0,2)$.
4. Find the values of $a$ and $b$ that make the function

$$
f(x)= \begin{cases}a x^{2}+x+1, & \text { if } x \leq 1 \\ b x-1, & \text { if } x>1\end{cases}
$$

differentiable everywhere. Find $f^{\prime}(x)$.
5. If $f(x)=\sin (g(x))$, find $f^{\prime}(2)$ given that $g(2)=\frac{\pi}{3}$ and $g^{\prime}(2)=\frac{\pi}{4}$.
6. Find the derivative
(a) $f(x)=x^{2} \cot (3 x)$
(b) $f(x)=\frac{e^{\sqrt{x}}}{x+\sqrt{x}}$
(c) $f(x)=\tan ^{3}\left(e^{-x}+e x-x^{e}\right)$
(d) $f(x)=\left(\frac{x^{3}+3 x}{x^{2}-4 x+1}\right)^{5 / 2}$
7. The vector function $\mathbf{r}(t)=<t+e^{4 t},-t \cos (2 t)>, 0 \leq t \leq 2 \pi$, represents the position of a particle at time $t$. Find the velocity acceleration vectors of the object at $t=\frac{\pi}{4}$.
8. At what point(s) does the curve parametrized by $x=t^{2}-6 t+5, y=t^{2}+4 t+3$ have a horizontal or vertical tangent?

