1. Find the area inside the curve $r = -2\sin\theta$ in the third quadrant. At what angles will the above curve intersect with the polar curve r = 1?

2. Find the area of the region enclosed by one loop of the curve $r = 4\cos 3\theta$.

3. Find the area of the region inside the curve $r = 3\cos\theta$ and outside the curve $r = 1 + \cos\theta$.

---

4. Evaluate the integral

   (a) $\displaystyle\int t^2 \cos(1 - t^3)dt$

   (b) $\displaystyle\int \frac{x^2}{\sqrt{1-x}}dx$

   (c) $\displaystyle\int x^3 e^{x^2} dx$

   (d) $\displaystyle\int (x^3 + 2x^2 - x)e^{3x}\ dx$

   (e) $\displaystyle\int \frac{\ln x}{x^2}dx$

   (f) $\displaystyle\int e^{3x} \sin(2x)\ dx$

   (g) $\displaystyle\int_0^{\pi/8} \sin^2(2x)\cos^3(2x)\ dx$

   (h) $\displaystyle\int \sin^2 x \cos^4 x\ dx$

   (i) $\displaystyle\int_0^{\pi/4} \tan^4 x \sec^4 x\ dx$

   (j) $\displaystyle\int \tan^3 x \sec^3 x\ dx$

   (k) $\displaystyle\int (4x^2 - 25)^{-3/2}\,dx$

   (l) $\displaystyle\int \frac{(x-1)^2}{5\sqrt{24 - x^2 + 2x}}dx$

   (m) $\displaystyle\int \frac{5x^2 + x + 12}{x^3 + 4x}dx$

5. Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$.

   (a) Find the area fo $\mathcal{R}$

   (b) Find the volume obtained by rotating $\mathcal{R}$ about the line $x = -1$.

   (c) Find the volume obtained by rotating $\mathcal{R}$ about the line $y = 2$.

6. Find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = x^2$ about

   (a) the line $y = -1$

   (b) the y-axis

(c) the line $x = 4$

7. The base of solid S is the triangular region with vertices (0,0), (2,0), and (0,1). Cross-sections perpendicular to the x-axis are semicircles. Find the volume of S.

8. A cable 40 feet long weighing 6 pounds per foot is hanging off the side of a 50 foot tall building. At the bottom of the cable is a bucket of rocks weighing 100 pounds. How much work is required to pull 10 feet of the cable to the top of the building?

9. A spring has a natural length of 20 cm. If a 10 J work is required to keep it stretched to a length 25 cm, how much work is done in stretching the spring from 30 cm to 80 cm?

10. A tank of water is 20 ft long and has a vertical cross section in a shape of an equilateral triangle with sides 2 ft long. The tank is filled with water to a depth of 18 inches. Determine the amount of work needed to pump all of the water to the top of the tank. The weight of water is 62.5 lb/ft^3 .

11. Find the average value of $f = \sin^2 x \cos x$ on $[-\pi/2, \pi/4]$.

12. Write out the form of the partial fraction decomposition (do not try to solve)

$$\frac{20x^3 + 12x^2 + x}{(x^3 - x)(x^3 + 2x^2 - 3x)(x^2 + x + 1)(x^2 + 9)^2}$$

13. Determine whether the given integral is convergent or divergent.

(a) $\displaystyle\int_1^\infty \frac{4 + \cos^4 x}{x} dx$

(b) $\displaystyle\int_1^\infty \frac{3 + \sin x}{x^2} dx$

(c) $\displaystyle\int_0^\infty \frac{1}{\sqrt{x} + e^{4x}} dx$

14. Compute the following integrals or show that they diverge.

(a) $\displaystyle\int_e^\infty \frac{dx}{x \ln^5 x}$

(b) $\displaystyle\int_{-\infty}^0 (1 + x)e^x \, dx$

(c) $\displaystyle\int_{-\infty}^\infty \frac{6x^5}{(x^6 + 3)^3} dx$

(d) $\displaystyle\int_0^{2020} \frac{1}{\sqrt{2020 - x}} dx$

15. Find the following limits

(a) $\displaystyle\lim_{n\to\infty} \frac{(-1)^n}{n^3}$

(b) $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}}{\ln n}$

(c) $\lim_{n\to\infty} \dfrac{1-2n^2}{\sqrt[3]{n^6+1}+2n^2}$

(d) $\lim_{n\to\infty} \left(\dfrac{1}{3}\ln(n^3+5n-2)-\ln(2n-1)\right)$

16. Show that the sequence defined by $a_1 = 3$ and $a_{n+1} = 6 - \dfrac{8}{a_n}$ is increasing and bounded above. Find its limit.

17. If the series $\sum_{n=1}^{\infty} a_n$ has a partial sum of $s_n = \dfrac{n}{2n+1}$, find a_4 and the sum of the series.

18. Find the sum of the series

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n+1}}{3^{3n-1}}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2-4}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{5^n n!}$

19. Which of the following series is convergent?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^{5/7}+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{3^n}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

20. Which of the following series is absolutely convergent?

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{n!}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n}{\sqrt{n-2}}$

(e) $\displaystyle\sum_{n=0}^{\infty} (-1)^n\frac{2^{2n}}{3^{3n}}$

21. Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$.

22. Find a power series centered at $x = 0$ for the given function function and determine the radius of convergence.

(a) $f(x) = \dfrac{x}{1-8x^3}$

(b) $f(x) = \ln(3 - 2x)$

(c) $f(x) = \dfrac{x^2}{(1 + 9x)^3}$

23. Find the Taylor series for the function $f(x) = \sqrt{x}$ at $a = 16$.

24. Find the Maclaurin series for the function $f(x) = x^2 \ln(1 + x^3)$.

25. Evaluate the integral $\displaystyle\int_0^{1/3} \frac{1}{1 + x^7} dx$ as an infinite series.

26. Find the length of the curve $x(t) = 3t - t^3$, $y(t) = 3t^2$, $0 \le t \le 2$.

27. Find the area of the surface obtained by rotating the curve $y = x^3$, $0 \le x \le 2$ about the x-axis.

28. Find the area of the surface obtained by rotating the curve $x = \sqrt{2y - y^2}$, $0 \le y \le 1$ about the y-axis.