Week in Review 3

Monday, February 5, 2024 5:28 PM

Exam 1 Review over Chapters 1 and 2

- Basic Matrix Operations
- Matrix Multiplication
- Review of Lines
- Modeling with Linear Functions
- Systems of Two Equations in Two Unknowns
- Setting Up and Solving Systems of Linear Equations

Pr 1. State the dimensions of the matrix $A = \overbrace{\begin{bmatrix} -2 & 4 & w \\ -2 & 9 & 13 \\ 3y & 0 & 8 \\ 0 & -7 & -2 \end{bmatrix}}^{3 \text{ columns}}$. 4 rows

4x3

Pr 2. State the value of b_{32} given $B = \begin{bmatrix} 6 & 3x & -y \\ 4w & 2 & -9 \\ -2y & \boxed{0} & 1 \\ 3x & 7 & 12w \end{bmatrix}$.

$b_{32} = 0.$

Pr 3. If A is a 2×3 matrix, B is a 2×3 matrix, and C is a 3×2 matrix, determine the size of $(2A+3B)^T - 25C$, if possible.

$(2A + 3B)^T$

2x3 2x3

2x3 $\xrightarrow{\text{Transpose}}$ 3x2

$\underbrace{(2A+3B)^T}_{3\times2}$ $\underbrace{-25C}_{3\times2}$

The size of $(2A+3B)^T - 25C$ is 3x2.

$A - B^T \neq (A-B)^T$

Pr 4. Determine the value of w, x, and y given $\begin{bmatrix} 2 & w-1 \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y & -6 \\ -8 & 12 \end{bmatrix}^T = 2\begin{bmatrix} -1 & 9 \\ 4 & -4 \end{bmatrix}$

$$\begin{bmatrix} 2 & w-1 \\ 2 & 4x \end{bmatrix} - \begin{bmatrix} y & -8 \\ -6 & 12 \end{bmatrix} = \begin{bmatrix} -2 & 18 \\ 8 & -8 \end{bmatrix}$$

$2\cdot(-1)$, $2\cdot 9$, $2\cdot 4$, $2\cdot(-4)$

$$\begin{bmatrix} 2-y & w-1+8 \\ 2+6 & \boxed{4x-12} \end{bmatrix} = \begin{bmatrix} -2 & 18 \\ 8 & \boxed{-8} \end{bmatrix}$$

1) $2 - y = -2$
2) $w - 1 + 8 = 18$
$w + 7 = 18$
3) $8 = 8$ ✓
4) $4x - 12 = -8$

1) $\longrightarrow$ $2 = -2 + y \Rightarrow \boxed{y = 4}$
$+2 \quad +2$

2) $\Rightarrow$ $w + 7 = 18$ $\boxed{w = 11}$
$-7 \quad -7$

4) $4x - 12 = -8$
$+12 \quad +12$
$\frac{4x}{4} = \frac{4}{4}$ $\boxed{x = 1}$

$\boxed{\begin{array}{l} w = 11 \\ x = 1 \\ y = 4 \end{array}}$

Pr 5. There are three food trucks in town which sell chicken. Last week, the east store sold 120 chicken fingers, 48 baskets of fries, 60 chicken sandwiches, and 60 cans of soda. The west store sold 105 chicken fingers, 72 baskets of fries, 21 chicken sandwiches, and 147 cans of soda. The north store sold 60 chicken fingers, 40 baskets of fries, 50 cans of soda, but no chicken sandwiches. Use a 4×3 matrix to express the sales information for these three food trucks last week. Then if sales at the food trucks are expected to decrease by 18% next week, use a matrix to show the expected sales for next week.

4 rows, three columns

$$\begin{array}{l} \\ \text{Fingers} \\ \text{Fries} \\ \text{Sandwiches} \\ \text{Soda} \end{array} \begin{array}{c} \begin{array}{ccc} E & W & N \end{array} \\ \begin{bmatrix} 120 & 105 & 60 \\ 48 & 72 & 40 \\ 60 & 21 & 0 \\ 60 & 147 & 50 \end{bmatrix} \end{array} = M$$

scaling changes all entries by the same factor

$.18M \overset{?}{=}$ decrease ☹

$M - .18M = (1 - .18)M = .82M$

$.82M$?

$(CAB)^T \neq CAB^T$

Pr 6. If A is a 2×4 matrix, B is a 2×4 matrix, and C is a 3×2 matrix, determine the size of CAB^T, if possible.

$$.82M = \begin{bmatrix} 98.4 & 86.1 & 49.2 \\ 39.36 & 59.04 & 32.8 \\ 49.2 & 17.22 & 0 \\ 49.2 & 120.54 & 41 \end{bmatrix}$$

exact Answer =

and round?

$\boxed{C \; A \; B^T \text{ has size } 3 \times 2}$

$\underset{3\times 2}{C} \; \underset{2\times 4}{A} \; \underset{4\times 2}{B^T}$

$\underset{3\times 4}{CA} \; \underset{4\times 2}{B^T}$

$\boxed{3 \times 2}$

If you tried $(CAB)^T$ not possible

Pr 7. Compute $\begin{array}{c} \text{red} \\ \begin{bmatrix} -2 & 3x & 3 \\ 6w & 0 & 2y \end{bmatrix} \end{array} \begin{array}{c} \text{blue} \\ \begin{bmatrix} -6 & 3m \\ 3n & 4 \\ -p & 0 \end{bmatrix} \end{array}$. :) ✓

2 × 3   3 × 2   result is size 2×2

$$= \begin{bmatrix} -2\cdot(-6) + 3x(3n) + 3(-p) & -2(3m) + 3x\cdot 4 + 3\cdot 0 \\ (6w)(-6) + 0(3n) + (2y)(-p) & (6w)(3m) + 0\cdot 4 + (2y)0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 + 9xn - 3p & -6m + 12x \\ -36w - 2yp & 18wm \end{bmatrix}$$ ✓

Pr 8. There are three food trucks in town which sell chicken. Last week, the east store sold 120 chicken fingers, 48 baskets of fries, 60 chicken sandwiches, and 60 cans of soda. The west store sold 105 chicken fingers, 72 baskets of fries, 21 chicken sandwiches, and 147 cans of soda. The north store sold 60 chicken fingers, 40 baskets of fries, 50 cans of soda, but no chicken sandwiches. Use a 4 × 3 matrix to express the sales information for these three food trucks last week. If all three trucks sell chicken fingers for \$1.50, a basket of fries for \$1, a can of soda for \$.50, and a chicken sandwich for \$2, how much did each food truck bring in last week?

E W N

4×3 matrix M

$M \cdot C =$ $\begin{bmatrix} \$ \\ \$ \\ \$ \end{bmatrix}$ E W N

4×3 × 3×1 → 4×1 :(

$M^T \cdot C$

3×4 4×1 → 3×1

$M^T \cdot$ [ ]

Fin. Fri. San. Soda

$$M^T = \begin{bmatrix} & \end{bmatrix} \cdot \begin{bmatrix} 1.50 \\ 1 \\ 2 \\ .50 \end{bmatrix} = \begin{bmatrix} \$378 \\ \$345 \\ \$155 \end{bmatrix}$$ ✓ ← north

Pr 9. Write the equation of the line that passes through the point $(-3, 7)$ and has a slope of zero. → horizontal ✓

y = constant

$$\boxed{y = 7}$$

Pr 10. You have a line which passes through the points $(3,-4)$ and $\left(\frac{1}{2},\frac{2}{3}\right)$. If x decreases by 6 units, what is the corresponding change in y?

slope $m = \frac{\Delta y}{\Delta x}$ $\Delta x = -6$

$-3 = \frac{-6}{2}$

$m = \frac{\frac{2}{3} - (-4)}{\frac{1}{2} - 3} = \frac{\frac{2}{3} + 4}{\frac{1}{2} - \frac{6}{2}} = \frac{\frac{2}{3} + \frac{12}{3}}{\frac{-5}{2}} = \frac{\frac{14}{3}}{\frac{-5}{2}} = \frac{14}{3} \cdot \frac{2}{-5} = \frac{28}{-15} = -\frac{28}{15}$

$\frac{\Delta y}{\Delta x} = \frac{-28}{15}$

$\frac{\Delta y}{-6} = \frac{-28}{15}$

$\Delta y = \frac{-28}{15} \cdot (-6) = \frac{56}{5}$

y increases by $\frac{56}{5}$ units or 11.2 units

Pr 11. An automobile purchased for use by the manager of a firm at a price of \$23,950 is to be depreciated using a linear model over ten years. What will the book value of the automobile be at the end of five years if the automobile has a scrap value of \$1,000 at the end of 10 years?

$V(t) = mt + b$

$(0, 23950)$, $(10, 1000)$

$m = \frac{1000 - 23950}{10 - 0} = \frac{-22950}{10} = -2295$

$b = V(0)$
$= m \cdot 0 + b$
$b = V(0) = 23950$

$V(t) = -2295t + 23950$

$V(5) = -2295 \cdot 5 + 23950$
$= -11475 + 23950$
$= \boxed{\$12{,}475}$

$23950 - 11475 = 12475$

Pr 12. Dave sells lemonade at his lemonade stand. He makes the lemonade for \$.10 per cup. When he sells 20 cups in a day, then his profit is \$15. When he sells 30 cups in a day, then his cost for that day is \$18.

(a) Determine the linear cost function.

$C(x) = mx + F$ ← is the fixed costs

m ← cost per unit

$m = .10$

$\boxed{C(x) = .10x + 15}$

$18 = C(30) = .10(30) + F$

$18 = 3 + F$
$-3 \quad -3$
$15 = F$

(b) Determine the linear revenue function.

$R(x) = ?$ $R(x) = P(x) + C(x)$ Do c first

$R(x) = 1.5x - 15 + .10x + 15$
$= 1.5x + .1x = 1.6x$

$\boxed{R(x) = 1.6x}$

(c) Determine the linear profit function. do c first

$P(x) = R(x) - C(x)$

$P(x) = mx + b$

$15 = P(20) = m(20) + b$

$b = -F = -15$

$15 = 20m - 15$
$+15 \quad +15$

$\frac{30}{20} = \frac{20m}{20}$

$m = \frac{3}{2} = 1.5$

$P(x) = 1.5x - 15$

(d) Determine and interpret the break-even point.

want $C(x) = R(x)$ $(x, R(x))$

$.10x + 15 = 1.6x$
$-.10x \quad -.1x$
$\frac{15}{1.5} = \frac{1.5x}{1.5}$

$x = \frac{150}{15} = 10$ cups

$R(10) = 1.6 \times 10 = \$16$

$\boxed{(10, \$16)}$

interpretation: If Dave sells 10 cups of lemonade, then his revenue of \$16 will match the cost of \$16.

2000

Pr 13. When a company sells a smartphone at $500, they manage to sell 2 thousand phones daily. If the price increases to $600, then the company sells 1600 phones daily. The company decides to use a new producer. The producer is willing to provide $\boxed{1500}$ phones if the price is $\boxed{\$250}$ and are willing to provide $\boxed{2100}$ phones when the price is $\boxed{\$350}$. Assume supply and demand are linear.

(a) Determine the linear $\boxed{\text{supply}}$ equation. "producer"

price $= p(x)$ ← rise $x =$ quantity ← run

$$m = \frac{350 - 250}{2100 - 1500} = \frac{100}{600} = \frac{1}{6}$$

$p(x) = mx + b$

$$250 = p(1500) = \frac{1}{6}(1500) + b$$

$$250 = \frac{1}{6} \cdot 1500 + b$$

$250 = 250 + b$, so $b = 0$

$$\boxed{p(x) = \frac{1}{6}x} \leftarrow \text{supply}$$

(b) Determine the linear demand equation.

price $= p(x) = mx + b$ "customers"

$$m = \frac{600 - 500}{1600 - 2000} = \frac{100}{-400} = -\frac{1}{4}$$

$$500 = p(2000) = -\frac{1}{4}(2000) + b$$

$$500 = -\frac{1}{4} \cdot 2000 + b$$

$$500 = -500 + b$$

$+500 \quad +500$

$b = 1000$

$$\boxed{p(x) = -\frac{1}{4}x + 1000}$$

(c) Determine and interpret the equilibrium point.

supply = demand

$$6 \times \left(\frac{1}{6}x\right) = \left(-\frac{1}{4}x + 1000\right) \times 6 \rightarrow 4(x) = \left(-\frac{6}{4}x + 6000\right)4$$

$$4x = -6x + 24000$$

$+6x \quad +6x$

$$\frac{10x}{10} = \frac{24000}{10}$$

$x = 2400$ phones

Supply

$$p(2400) = \frac{1}{6} \cdot 2400 = \$400$$

If we sell phones at $400, then the producer will produce 2400 phones, all of which will be sold.

Pr 14. Determine the value of k so that the following system of linear equations has exactly one solution.

$$-x + ky = 24 \rightarrow -x = -ky + 24$$
$$3x - 6y = 30$$

$x = ky - 24$ ← Equation 1

substitute $x = ky - 24$ into Eq. 2

$$3(ky - 24) - 6y = 30$$

$$3ky - 3 \cdot 24 - 6y = 30$$

$$3ky - 6y - 72 = 30$$

$+72 \quad +72$

$$\frac{(3k - 6)y}{3k - 6} = \frac{102}{3k - 6}$$

$$y = \frac{102}{3k - 6}$$

unique solution, provided $3k - 6 \neq 0$.

need $3k - 6 \neq 0$

$3k \neq 6$

$\boxed{\text{need } k \neq 2}$

Pr 15. Solve the following system using substitution.

$$-2x + 10y = 90$$
$$7x + 4y = 75$$

$$-2x + 10y = 90$$

$+2x \quad +2x$

$$\frac{10y}{10} = \frac{90 + 2x}{10}$$

$$y = \frac{90}{10} + \frac{2x}{10} = 9 + \frac{x}{5} \rightarrow y = 9 + \frac{5}{5}$$

$y = 9 + 1 = 10$

$$7x + 4\left(9 + \frac{x}{5}\right) = 75$$

$$7x + 4 \cdot 9 + \frac{4x}{5} = 75$$

$$7x + 36 + \frac{4x}{5} = 75$$

$-36 \quad -36$

$$7x + \frac{4x}{5} = 39$$

$$\frac{35x}{5} + \frac{4x}{5} = 39$$

$$\frac{5}{39} \cdot \frac{39x}{5} = 39 \cdot \frac{5}{39}$$

$\boxed{x = 5}$

$\boxed{y = 10}$

Pr 16. Solve the following system using the addition method.

$$x - 3y = 4$$
$$-\frac{1}{6}x + \frac{1}{2}y = \frac{2}{3}$$

Eq1 + 6 · Eq 2

$x - 3y = 4$

$+\,6\left(-\frac{1}{6}x + \frac{1}{2}y = \frac{2}{3}\right)$

$x - 3y = 4$

$-x + 3y = 4$

$0 + 0 = 8$

$0 = 8$

no solutions

Pr 17. Set up and solve the following problem as a system of linear equations.

Donald has \$18,000 to invest. He decides to invest in three different companies. The Huey company costs \$260 per share and pays dividends of \$3 per share each year. The Dewey company costs \$75 per share and pay dividends of \$1.00 per share each year. The Louie company costs \$90 per share and pays \$2.00 per share per year in dividends. 1) Link wants to have twice as much money in the Dewey company as in the Louie company. 2) Link also wants to earn \$220 in dividends per year. How much should Link invest in each company to meet his goals?

1) twice as much in Dewey (D) as in Louie (L)

H = # of shares of Huey stock

D = # of sh... of Dewey stock

L = # of " of Louie stock

2D = L or D = 2L ?

~~L=1~~, D = 2

2) $3H + 1D + 2L = 220$

3) $260H + 75D + 90L = 18000$

I Set up the system of equations.

$D - 2L = 0$

$3H + D + 2L = 220$

$260H + 75D + 90L = 18000$

How to solve? matrices on calc.

H = 60 shares

D = 20 shares

L = 10 shares

Pr 18. Write the augmented matrix corresponding to the given system of linear equations.

$$x - 5y = 4$$
$$-3x + 2y - 7z = -5$$
$$-2y + 4 = 3z$$

$x - 5y + 0z = 4$

$-3x + 2y - 7z = -5$

$0x - 2y - 3z = -4$

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -5 & 0 & 4 \\ -3 & 2 & -7 & -5 \\ 0 & -2 & -3 & -4 \end{array}\right]$$

Pr 19. Determine if the augmented matrix is in reduced row-echelon form or not.

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & -4 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Yes ✓

Pr 20. Perform the following row operations $2R_1 + R_2 \to R_2$ and $-R_1 + R_3 \to R_3$, in order on the given matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ -1 & 1 & 1 & 3 \\ 2 & 2 & -3 & 8 \end{array}\right]$$

↑ come back

$2R_1 \quad 2(1 \quad 0 \quad 2 \quad 7) \rightarrow \quad 2 \quad 0 \quad 4 \quad 14$

$+ R_2 \quad + \quad -1 \quad 1 \quad 1 \quad 3 \qquad -1 \quad 1 \quad 1 \quad 3$

$1 \quad 1 \quad 5 \quad 17$

$$A \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 1 & 1 & 5 & 17 \\ 2 & 2 & -3 & 8 \end{array}\right]$$

$-R_1 + R_3$

$-1 \quad 0 \quad -2 \quad -7$

$+ \; 2 \quad 2 \quad -3 \quad 8$

$1 \quad 2 \quad -5 \quad 1$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 1 & 1 & 5 & 17 \\ 1 & 2 & -5 & 1 \end{array}\right]$$

Pr 21. Use ~~row operations~~ calculator to transform the matrix into reduced row-echelon form.

$$A = \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 3 & 2 & 12 \end{array}\right]$$

$$\text{rref}(A) = \left[\begin{array}{cc|c} 1 & 0 & 17/4 \\ 0 & 1 & -3/8 \end{array}\right] \checkmark$$

↑ I used the calculator

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 3 & 2 & 12 \end{array}\right] \xrightarrow{-3R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 8 & -3 \end{array}\right]$$

$$\xrightarrow{\frac{1}{8}R_3 \to R_3} \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -3/8 \end{array}\right] \xrightarrow{R_1 + 2R_2 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & 17/4 \\ 0 & 1 & -3/8 \end{array}\right]$$

Pr 22. Solve the system of linear equations, using technology.

$$\begin{aligned} y &= x - 2 \\ y - z &= 1 \\ x + z &= 3 \end{aligned}$$

matrix software

Ax + By + Cz = const

$-x + y + 0z = -2$

$0x + y - z = 1$

$x + 0y + z = 3$

$$\rightarrow \begin{array}{ccc|c} x & y & z & \\ \end{array} \left[\begin{array}{ccc|c} -1 & 1 & 0 & -2 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{\text{rref}} \begin{array}{ccc|c} x & y & z & \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\boxed{\begin{aligned} x &= 3 \\ y &= 1 \\ z &= 0 \end{aligned}}$$

Pr 23. Solve the system of linear equations, using technology.

$$\begin{aligned} 2x + 2y - 4z &= 24 \\ x + z &= -9 \\ -x - y + 2z &= -12 \end{aligned}$$

$$\rightarrow \begin{array}{ccc|c} x & y & z & \\ \end{array} \left[\begin{array}{ccc|c} 2 & 2 & -4 & 24 \\ 1 & 0 & 1 & -9 \\ -1 & -1 & 2 & -12 \end{array}\right]$$

$$\text{rref}(A) = \left[\begin{array}{ccc|c} 1 & 0 & 1 & -9 \\ 0 & 1 & -3 & 21 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left.\begin{aligned} x + z &= -9 \\ y - 3z &= 21 \end{aligned}\right\} \quad \begin{aligned} x &= -z - 9 \\ y &= 3z + 21 \end{aligned}$$

$x = -t - 9$

$y = 3t + 21$

$z = t$

$\{ (-t-9, 3t+21, t) : t \text{ is any real number} \}$

one situation

suppose $\text{rref}(A) = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]$

no solution

$0 = 1$

Pr 24. Assume your solution to a real-world application problem was $(x, y, z) = (5 - t, -6 + 2t, t)$. If x, y, and z represent the number of whole items produced, how many solutions does the problem actually have?

1) $5 - t \geq 0$

2) $-6 + 2t \geq 0$

3) $t \geq 0$

1) $5 \geq t \rightarrow t = \cancel{0}, \cancel{1}, \cancel{2}, 3, 4, 5$

2) $2t \geq 6 \rightarrow \boxed{t \geq 3}$

There are only three solutions