2024 Fall Math 140 Week-In-Review

Week 11: Sections 5.7 and 5.8

Some Key Words and Terms: Function Transformations, Function Arithmetic, Function Composition, One-to-One Functions, Exponential and Logarithmic Form, Properties of Logarithms, Logarithmic Function, Solving Exponential and Logarithmic Equations, Exponential Models.

Function Transformations:

Parent Function	Horizontal Shift	Vertical Shift	Reflection across x-axis	Vertical Expansion	Vertical Compression
$f(x)$	$f(x+a)$	$f(x)+a$	$-f(x)$	$af(x)$	$\frac{1}{a}f(x)$
x^2					
x^3					
$\sqrt{x}$					
$\sqrt[3]{x}$					
$|x|$					
b^x					
$\left(\frac{1}{b}\right)^x$					

Function Arithmetic:

Function Composition:

One-to-One Functions:

Exponential and Logarithmic Form:

Properties of Logarithms:

Logarithmic Function:

Solving Exponential and Logarithmic Equations:

Exponential Models:

Examples:

1. For the given functions, state the parent function. Then, state the transformations performed on the parent function to obtain the given function.

   (a) $g(x) = 5\sqrt[3]{x+1} + 2$

   (b) $h(x) = -\dfrac{2}{3} \cdot \left(\dfrac{1}{7}\right)^{x-4}$

2. For the parent function $f(x) = x^2$, write a function $k(x)$ which is $f(x)$ with the following transformations:

   - shift down 9 units
   - vertically compress by a factor of 3
   - shift left 2 units
   - reflect across the x-axis

3. For the given functions, compute the indicated value, if it exists..

$$f(x) = 5x - x^2 \qquad g(x) = \frac{3}{x} - 7 \qquad h(x) = 2\sqrt{10 - x}$$

(a) $(f + g)(2)$

(b) $\left(\dfrac{f}{h}\right)(-6)$

(c) $(g \circ f)(5)$

(d) $(h \circ f)(2)$

4. Convert the following from an exponential equation to a logarithmic equation.

   (a) $7^{3x} = 11$

   (b) $\left(\dfrac{5}{2}\right)^2 = x$

5. Convert the following from a logarithmic equation to an exponential equation.

   (a) $\log_9(x + 2) = -4$

   (b) $\ln(5) = x - 1$

6. Rewrite the following expression as a single logarithmic term.

   $7\log_5(x) + 2 - \log_5(x - 3) - 2\log_5(x - 1)$

7. Fully expand and simplify the given logarithmic expression.

$$\log_3\left(\frac{27x^7z^4}{w^5y^{11}}\right)$$

8. Determine the domain of the given functions in interval notation.

(a) $f(x) = \dfrac{\sqrt{11-2x}}{\sqrt[3]{x-1}+2}$

(b) $g(x) = \dfrac{x+5}{x-5} + 3\log_2(4x+20)$

9. Solve the given equations for x. Express your answer in exact form and in terms of the natural logarithm when necessary.

(a) $3^{2x} = 7^{x+1}$

(b) $7(2e^{8x} - 3) = -11$

(c) $2^{4x} + 3 \cdot 2^{2x} = 10$

10. Solve the given equations for x. Express your answer in exact form.

(a) $\log(x) - \log(x-2) = 1$

(b) $\ln(x+2) + \ln(x-3) = \ln(x) + \ln(x-5)$

(c) $\log_4(2x^2 + 2x) - \log_4(x+3) = \log_4(x+1)$

11. How much money would you need to deposit in a savings account that earns 4.5% annual interest compounded monthly if after 8 years, you want there to be $30,000 in the account?

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

12. A savings account grows from an initial investment of $4,500 to $6,800 in 4 years. Calculate the annual interest rate for the savings account if the interest is compounded continuously. Express your answer in exact form then express your answer as a decimal rounded to 3 decimal places.