Math 151
Week-In-Review 15

Final Exam Review
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Problem Statements

1. Two forces act on an object. One force has magnitude 40 N in the direction of 30° above the postive $x-$axis. The other has magnitude 10 in the direction of 45° above the negative $x-$axis.

   (a) Find the magnitude and direction of the resulting force.

   (b) Find the same resulting direction as a bearing.

   (c) If the object moves from the origin to the point $(4, 10)$ find the work done by the resulting force.

   (d) If a wind blows 5 mph at S60°E, find the components of the wind as a vector.

2. $\boldsymbol{a} = \langle 2, -5 \rangle$, $\boldsymbol{b} = \langle 3, 7 \rangle$, $\boldsymbol{c} = \langle 10, -2 \rangle$.

   (a) Find $2\boldsymbol{a} - 3\boldsymbol{b}$.

   (b) Find the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

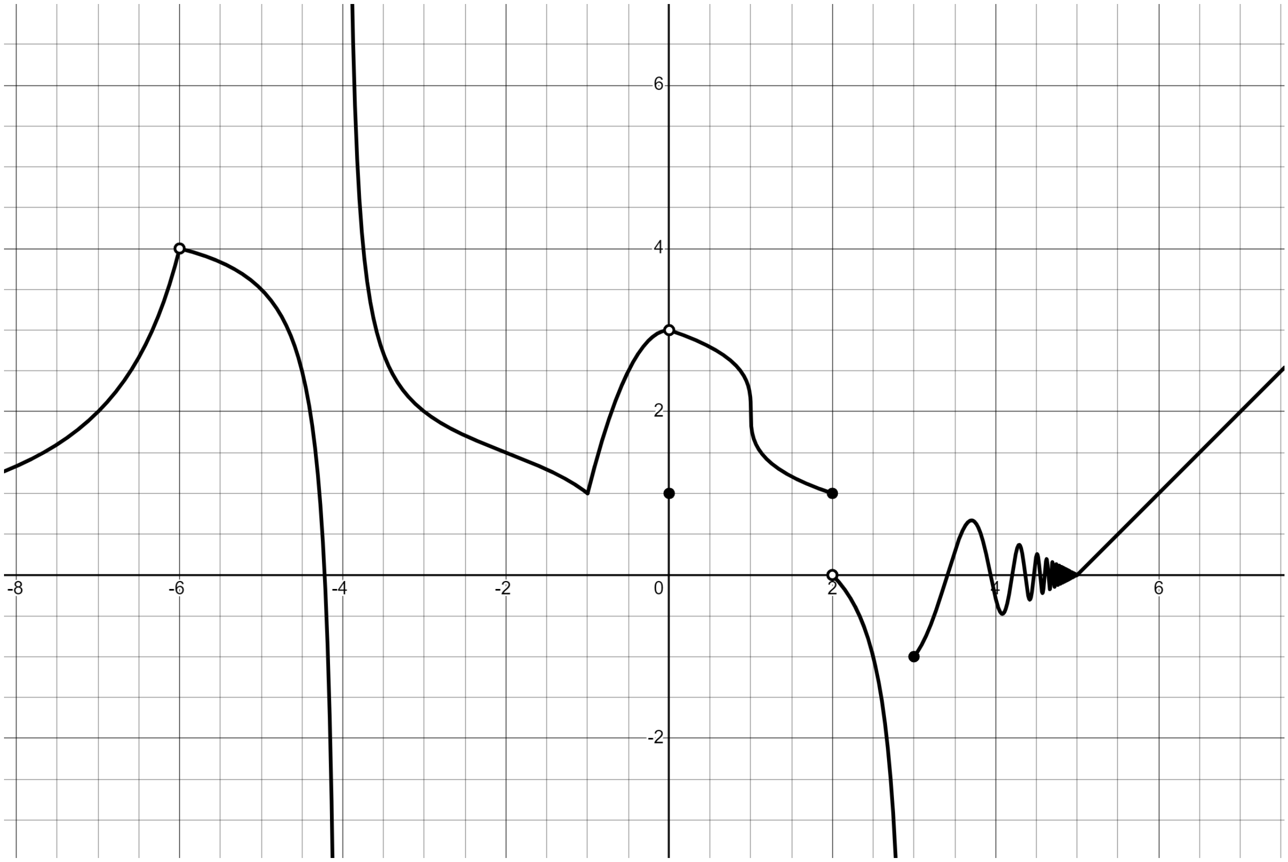
   (c) Find the magnitude of the vector projection of $\boldsymbol{c}$ onto $\boldsymbol{b}$.

3. Simplify the following:

   (a) $\arccos\left(\sin\left(\frac{11\pi}{6}\right)\right)$

   (b) $\sin(\arctan(x^2))$

4. Evaluate the limits for the graph of $f(x)$ below.



(a) $\lim\limits_{x\to -6^-} f(x) =$

(b) $\lim\limits_{x\to -6^+} f(x) =$

(c) $\lim\limits_{x\to -6} f(x) =$

(d) $\lim\limits_{x\to -4^-} f(x) =$

(e) $\lim\limits_{x\to -4^+} f(x) =$

(f) $\lim\limits_{x\to -4} f(x) =$

(g) $\lim\limits_{x\to 2^-} f(x) =$

(h) $\lim\limits_{x\to 2^+} f(x) =$

(i) $\lim\limits_{x\to 2} f(x) =$

5. For the graph above (ignoring $x = 5$),

(a) Where is $f(x)$ not continous?

(b) Where is $f(x)$ not differentiable?

6. Evaluate the limits.

(a) $\displaystyle\lim_{t\to 1} \frac{3t^2 + 2t - 5}{2t^2}$

(b) $\displaystyle\lim_{x\to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

(c) $\displaystyle\lim_{h\to 0} \frac{(x+h)^3 - 5(x+h)^2 - x^3 + 5x^2}{h}$

(d) $\displaystyle\lim_{x\to 0} \frac{\sin(x) + e^x - 2x - 1}{x^2}$

7. Consider the function $f(x) = 2\sqrt{x+5} - 4$

   (a) Find the average rate of change of the function on the interval $[4, 11]$.

   (b) Find c such that $f'(c)$ is equal to your answer from part (a).

   (c) Show there is a solution to $f(x) = 3$ on the interval $[4, 11]$.

   (d) Set up a limit representing the instantaneous rate of change of $f(x)$ when $x = 4$.

   (e) Find the instantaneous rate of change of $f(x)$ when $x = 4$.

8. Consider the piecewise function $f(x) = \begin{cases} 5 - \dfrac{1}{x} & \text{if} \quad x \leq 1 \\ \dfrac{x^2 - 1}{x - 1} & \text{if} \quad x > 1 \end{cases}$

   (a) Where is $f(x)$ not continuous?

   (b) Where is $f(x)$ not differentiable?

   (c) Find any vertical asymptotes of $f(x)$.

   (d) Find any horizontal asymptotes of $f(x)$.

9. Find $\frac{dy}{dx}$ using implicit differentiation.

$$\cos(xy) + \log_8(x) = e^{x+y} - y^5$$

10. Find $\frac{d}{dx}\left[x^{\arctan(x)}\right]$

11. A bacteria grows at a rate proportional to its size.

    (a) If there are 200 cells when $t = 4$ and 1000 cells when $t = 10$, find an equation representing the number of cells at time t.

    (b) When will there be 20000 cells?

12. A plane flies at an altitude of 5 km and passes over a a tracking telescope on the ground. When the angle of elevation of the telescope is $\frac{\pi}{3}$, the angle of elevation is decreasing at a rate of $\frac{\pi}{6}$. How fast is the plane traveling at that time?

13. (a) Use a linear approximation to estimate $\sqrt{630}$.

(b) The radius of a sphere was measured to be 20 cm with a possible error in measurement of 0.5 cm. What is the maximum possible error in using this value of the radius to calculate the volume of the sphere?

14. (a) Find the intervals where $f(x) = e^{x^2+6x}$ is increasing/decreasing.

(b) Determine the locations of any extreme values of $f(x)$.

(c) Find the intervals where $g(x) = x^4 - x^3 + 6x^2 - 15x + 10$ is concave up/concave down.

(d) Determine the locations of any inflection points of $g(x)$.

15. Consider the function $f(x) = x^2 + 5$.

    (a) Estimate the area under the function on the interval $[2, 8]$ using three equally-spaced rectangles and midpoints.

    (b) Set up an integral representing the exact area under the curve on the interval $[2, 8]$.

16. Evaluate

(a) $\displaystyle\int \left(\frac{1}{\sqrt{1-x^2}} + \sin(x) - 5x \right) \, dx$

(b) $\displaystyle\int_1^3 \frac{2^x}{2^x + 3} \, dx$

(c) $\displaystyle\frac{d}{dx}\left[\int_{\ln(x)}^{e^{4x}} t \cdot \sec(t) \, dt \right]$