Problem 1. Evaluate $\int_C (x^2 + y^2 + z^2)\, ds$, where C is parameterized by $\mathbf{r}(t) = \langle\ t,\ \cos 2t, \sin 2t\rangle$, $0 \le t \le 2\pi$.

Notice the differential, $ds = |r'(t)|\, dt$.

General formula : $\int_C f(x,y,z)\,ds = \int_a^b f(\vec{r}(t))\,|r'(t)|\,dt$,

where $C = \vec{r}(t)$, $a \le t \le b$

$r'(t) = \langle 1, -2\sin 2t, 2\cos 2t \rangle$

$|r'(t)| = \sqrt{1 + 4\sin^2 2t + 4\cos^2 2t} = \sqrt{5}$

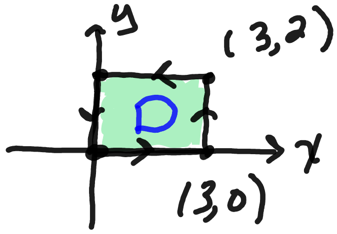
$$\int_C (x^2 + y^2 + z^2)\, ds = \int_0^{2\pi} (t^2 + \cos^2 2t + \sin^2 2t)\sqrt{5}\, dt$$

$$= \sqrt{5}\int_0^{2\pi} (t^2 + 1)\, dt$$

$$= \sqrt{5}\left(\frac{t^3}{3} + t\right)\Big|_0^{2\pi}$$

$$= \boxed{\sqrt{5}\left(\frac{8\pi^3}{3} + 2\pi\right)}$$

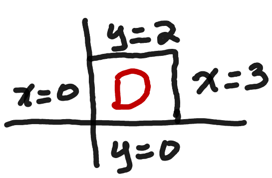
Problem 2. Evaluate $\int_C y^2\,dx + xy\,dy$, where C is the positively oriented rectangle with vertices $(0,0)$, $(3,0)$, $(3,2)$, and $(0,2)$.



- we do not want to solve this as a line integral since there are 4 lines.
- Notice C is a closed positively oriented curve, so we can use Green's Theorem.

$$\int_C P\,dx + Q\,dy = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA,$$ where D is the region bounded by C.

$$\int_C \underbrace{y^2}_{P}\,dx + \underbrace{xy}_{Q}\,dy = \iint_D (y - 2y)\,dA = \iint_D (-y)\,dA$$



D: $0 \le x \le 3$
$\quad\;\; 0 \le y \le 2$

$$= -\int_0^2 \int_0^3 y\,dx\,dy$$

$$= -\int_0^2 yx \Big|_{x=0}^{x=3} dy$$

$$= -\int_0^2 3y\,dy$$

$$= -\frac{3y^2}{2}\Big|_0^2 = \boxed{-6}$$

Problem 3. Given $\mathbf{F} = \langle 4xe^z, \cos(y), 2x^2e^z \rangle$ and $\mathbf{r}(t) = \langle \sin(t), t, \cos(t) \rangle$, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $0 \le t \le \frac{\pi}{2}$.

Hint: Find curl(F) to see if $\vec{F}$ is conservative.

$$\text{curl}F = \nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 4xe^z & \cos y & 2x^2e^z \end{vmatrix} = \langle 0, -(4xe^z - 4xe^z), 0 \rangle = \vec{0}$$

fundamental theorem of line integrals

$\vec{F}$ is conservative, so we can use FTLI

$$\int_C F\,dr = \int_C \nabla f \cdot dr = f\left(\vec{r}\left(\tfrac{\pi}{2}\right)\right) - f\left(\vec{r}(0)\right) = f\left(1, \tfrac{\pi}{2}, 0\right) - f(0,0,1),$$

where $f(x,y,z)$ is the potential function.

$$\vec{F} = \nabla f \rightarrow \langle 4xe^z, \cos y, 2x^2e^z \rangle = \langle f_x, f_y, f_z \rangle$$

$$f(x,y,z) = \int 4xe^z\,dx = 2x^2e^z + g(y,z)$$

$$f(x,y,z) = \int \cos y\,dy = \sin y + h(x,z)$$

$$f(x,y,z) = \int 2x^2e^z\,dz = 2x^2e^z + k(x,y)$$

$$f(x,y,z) = 2x^2e^z + \sin y$$

$$\int_C F \cdot dr = f\left(1, \tfrac{\pi}{2}, 0\right) - f(0,0,1)$$

$$= 2 + 1 - (0) = \boxed{3}$$

Problem 4. Evaluate $\int_C (x - z + y)\, ds$, where C is the line segment from $(2, 1, 1)$ to $(3, -1, 0)$.

Parameterize the line segment:

$\vec{r}(t) = \vec{r_0} + t\vec{v}$

$= \langle 2, 1, 1 \rangle + t\langle 1, -2, -1 \rangle, \quad \boxed{0 \le t \le 1}$

why? if $t=0$, do you get the initial point? If $t=1$, do you get the terminal point?

$\vec{r}(t) = \langle 2+t, 1-2t, 1-t \rangle, \quad 0 \le t \le 1$

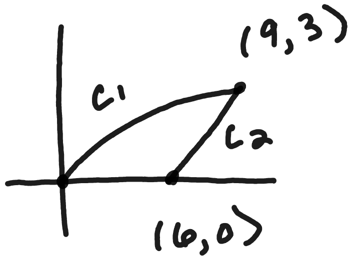
$\vec{r}\,'(t) = \langle 1, -2, -1 \rangle$ and $|\vec{r}\,'(t)| = \sqrt{6}$

$$\int_C (x - z + y)\, ds = \int_0^1 \left(2 + t - (1-t) + 1 - 2t\right)\sqrt{6}\, dt$$

$$= \sqrt{6}\int_0^1 2\, dt$$

$$= \boxed{2\sqrt{6}}$$

Problem 5. Set up but do not evaluate $\int_C xy\,dx + 2y\,dy$, where C is the arc of the curve $y = \sqrt{x}$ from $(0,0)$ to the point $(9,3)$, then the line segment from the point $(9,3)$ to the point $(6,0)$.



C_1. $y = \sqrt{x}$ Let $x = t$, $y = \sqrt{t}$, $0 \le t \le 9$

$dx = dt$, $dy = \frac{1}{2\sqrt{t}}\,dt$

C_2: Line from $(9,3)$ to $(6,0)$

$r(t) = \vec{r_0} + t\vec{v} = \langle 9,3\rangle + t\langle -3,-3\rangle$

$x = 9-3t$, $y = 3-3t$, $0 \le t \le 1$

$dx = -3\,dt$, $dy = -3\,dt$

$$\int_C xy\,dx + 2y\,dy = \int_{C_1} xy\,dx + 2y\,dy + \int_{C_2} xy\,dx + 2y\,dy$$

$$\boxed{\begin{gathered} = \int_0^9 \underbrace{(t)}_{x}\underbrace{(\sqrt{t})}_{y}\underbrace{(dt)}_{dx} + 2\underbrace{\sqrt{t}}_{y}\underbrace{\left(\frac{1}{2\sqrt{t}}\right) dt}_{dy} \\ + \\ \int_0^1 \underbrace{(9-3t)}_{x}\underbrace{(3-3t)}_{y}\underbrace{(-3\,dt)}_{dx} + 2\underbrace{(3-3t)}_{y}\underbrace{(-3\,dt)}_{dy} \end{gathered}}$$

Problem 6. Find the work done by the force field $\mathbf{F} = \langle x\cos y,\ y\rangle$ in moving a particle along the parabola $y = 2x^2$ from the point $(1, 2)$ to the point $(2, 8)$.

$\vec{F}$ is not conservative (why?)

therefore we must parameterize the curve.

Let $x = t$, then $y = 2t^2$, $1 \le t \le 2$

$\vec{r}(t) = \langle t, 2t^2 \rangle$, $r'(t) = \langle 1, 4t \rangle$

$$W = \int_C F \cdot dr = \int_1^2 \langle t\cos(2t^2), 2t^2 \rangle \cdot \langle 1, 4t \rangle \, dt$$

$$\int_1^2 \left(\underbrace{t\cos(2t^2)}_{\text{u-sub}} + 8t^3\right) dt$$

u-sub

$u = 2t^2$

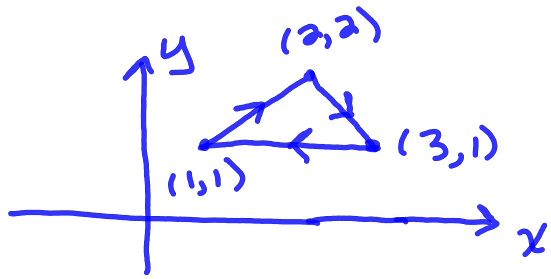
$du = 4t\, dt$

$$= \left(\frac{1}{4}\sin(2t^2) + 2t^4\right)\Big|_1^2$$

$$= \frac{1}{4}\sin(8) + 32 - \left(\frac{1}{4}\sin(2) + 2\right)$$

$$= \boxed{\frac{1}{4}\sin(8) - \frac{1}{4}\sin(2) + 30}$$

Problem 7. A particle is moving along a triangular path. The particle starts at the point $(1,1)$, then to the point $(2,2)$, then from $(2,2)$ to the point $(3,1)$, then back to the point $(1,1)$. Find the work done on this particle by the force field $\mathbf{F} = \langle x+1, y-2x \rangle$.

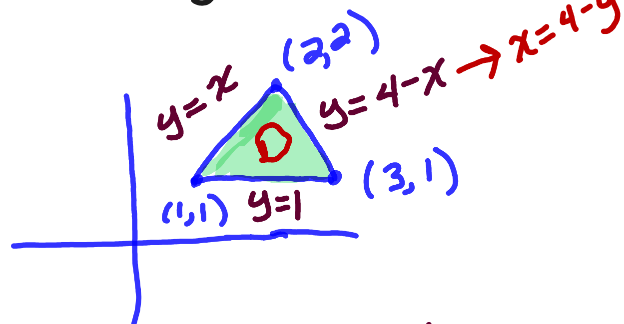


This is a closed curve oriented clockwise. We can use Green's theorem as follows:

$$W = \int_C F \cdot dr = -\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA, \quad P = x+1, \quad Q = y - 2x$$

$$= -\iint_D (-2 - 0)\, dA$$

$$= \iint_D 2\, dA$$



To prevent having two integrals, choose $dA = dx\,dy$

$D: \; 1 \le y \le 2$
$\quad y \le x \le 4-y$

$$\int_1^2 \int_y^{4-y} 2\, dx\, dy$$

$$\int_1^2 \left(2x \Big|_{x=y}^{x=4-y} \right) dy$$

$$\int_1^2 2(4-y-y)\, dy$$

$$\int_1^2 (8-4y)\, dy = \boxed{2}$$

set-up for $dA = dy\,dx$

For $1 \le x \le 2$, $1 \le y \le x$

For $2 \le x \le 3$, $1 \le y \le 4-x$

$$\int_1^2 \int_1^x 2\, dy\, dx + \int_2^3 \int_1^{4-x} 2\, dy\, dx$$

Problem 8. Evaluate $\int_C x\,dz + y\,dx + (xz)\,dy$, where C is parameterized by $\mathbf{r}(t) = \langle t^2, t^3, 2t\rangle$, $0 \le t \le 1$.

$x = t^2$ $\quad$ $y = t^3$ $\quad$ $z = 2t$

$dx = 2t\,dt$ $\quad$ $dy = 3t^2\,dt$ $\quad$ $dz = 2\,dt$

$$\int_C x\,dz + y\,dx + (xz)\,dy = \int_0^1 t^2 \cdot 2\,dt + t^3 \cdot 2t\,dt + (2t^3)3t^2\,dt$$

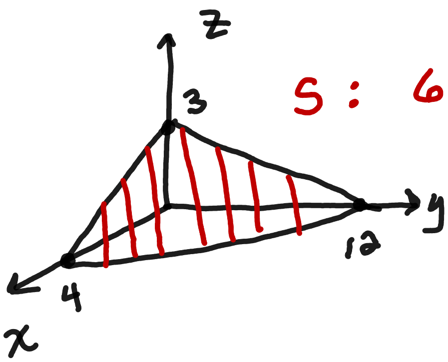
$$= \int_0^1 \left(2t^2 + 2t^4 + 6t^5\right) dt$$

$$= \left(\frac{2t^3}{3} + \frac{2t^5}{5} + t^6\right)\Bigg|_0^1$$

$$= \left(\frac{2}{3} + \frac{2}{5} + 1\right)\Bigg|_0^1$$

$$= \boxed{\frac{31}{15}}$$

Problem 9. Consider the part of the plane $6x + 2y + 8z = 24$ that lies in the first octant. Set up but **do not evaluate** a double integral that gives the surface area of this plane in the order $dA = dzdx$.

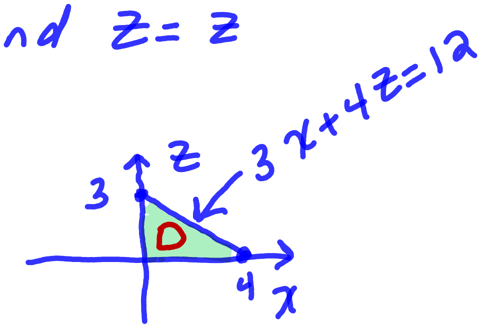


$S: \ 6x + 2y + 8z = 24.$

By solving this plane for y and choosing the parameters to be x and z, we will then be integrating in the xz-plane:

Let $x = x$, $y = \dfrac{24 - 6x - 8z}{2}$, and $z = z$

$\vec{r}(x, z) = \langle x, 12 - 3x - 4z, z \rangle$

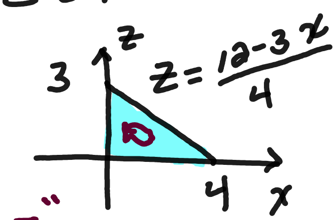


$A(S) = \iint_D |\vec{r}_x \times \vec{r}_z| \, dA$

$\vec{r}_x \times \vec{r}_z = \begin{vmatrix} i & j & k \\ 1 & -3 & 0 \\ 0 & -4 & 1 \end{vmatrix} = \langle -3, -1, -4 \rangle$

$A(S) = \iint_D \sqrt{26} \, dA.$ Now, we were told to choose $dA = dz\,dx$

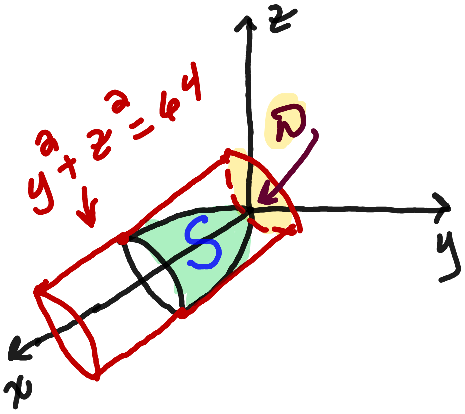
$$\boxed{A(S) = \int_0^4 \int_0^{\frac{12-3x}{4}} \sqrt{26} \, dz \, dx}$$



D as "Type I":

$0 \le x \le 4, \ 0 \le z \le \dfrac{12-3x}{4}$

Problem 10. Find the surface area of the part of the paraboloid $x = 4y^2 + 4z^2$ that lies inside the cylinder $y^2 + z^2 = 64$.



parameterize S:

let $y = y$, $z = z$, $x = 4y^2 + 4z^2$

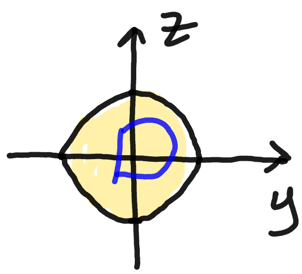
$\vec{r}(y,z) = \langle 4y^2 + 4z^2, y, z \rangle$, $y^2 + z^2 \le 64$

$$r_y \times r_z = \begin{vmatrix} i & j & k \\ 8y & 1 & 0 \\ 8z & 0 & 1 \end{vmatrix}$$

$$= \langle 1, -8y, -8z \rangle$$

$$A(S) = \iint_{y^2+z^2\le 64} |r_y \times r_z| \, dA = \iint_{y^2+z^2 \le 64} \sqrt{1+64y^2+64z^2} \, dA$$

D is polar in the yz-plane:



D : $0 \le \theta \le 2\pi$
$0 \le r \le 8$

$$A(S) = \int_0^{2\pi} \int_0^8 \sqrt{1+64r^2} \, r \, dr \, d\theta$$

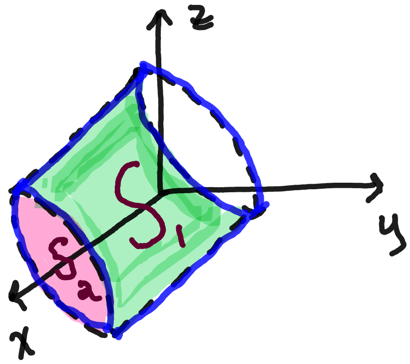
u-sub, $u = 1+64r^2$
$du = 128 r \, dr$

$$A(S) = \int_0^{2\pi} d\theta \int_0^8 r\sqrt{1+64r^2} \, dr$$

sorry for huge number :)

$$= \theta \Big|_0^{2\pi} \frac{1}{128} \frac{2}{3} (1+64r^2)^{3/2} \Big|_0^8 = (2\pi) \frac{1}{192} \left((4097)^{\frac{3}{2}} - 1 \right)$$

Problem 11. Consider the surface S that is the part of the the cylinder $y^2 + z^2 = 9$, $0 \le x \le 2$, including the disk $x = 2$. Find a parameterization of S.



S_1: The part of the cylinder

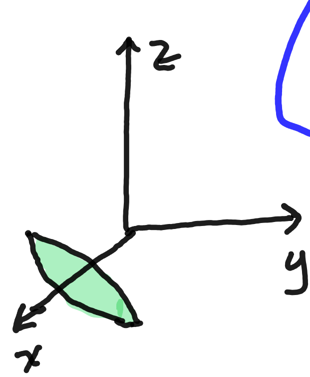
$y^2 + z^2 = 9$, $0 \le x \le 2$:

$\vec{r}_1(\theta, x) = \langle x, 3\cos\theta, 3\sin\theta \rangle$,

where $0 \le x \le 2$ and $0 \le \theta \le 2\pi$

since we are *on* the cylinder, $r = 3$

S_2: The disk on the plane $x = 2$ that is inside the cylinder $y^2 + z^2 = 9$:



S_2 is the disk $y^2 + z^2 \le 9$ that lies in the $x = 2$ plane.

$\vec{r}_2(r, \theta) = \langle 2, r\cos\theta, r\sin\theta \rangle$

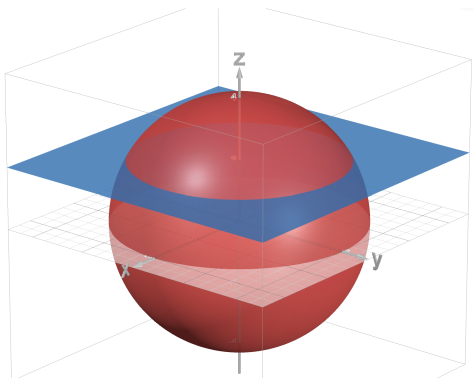
$0 \le r \le 3$, $0 \le \theta \le 2\pi$

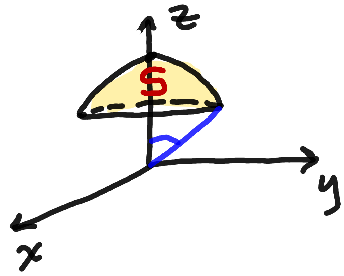
Here, since we are inside a circle, r = variable

$\boxed{S \text{ is the union of } S_1 \text{ and } S_2}$

Problem 12. Evaluate $\iint_S z\,dS$ where S is the part of the sphere $x^2+y^2+z^2=16$ that lies above the plane $z=2$.

Recall: A sphere of radius ρ is parameterized by $\mathbf{r}(\phi,\theta)=\langle \rho\sin(\phi)\cos(\theta),\ \rho\sin(\phi)\sin(\theta),\ \rho\cos(\phi)\rangle$, where $0\le\theta\le 2\pi$ and $0\le\phi\le\pi$. Furthermore, $|\mathbf{r}_\phi\times\mathbf{r}_\theta|=\rho^2\sin(\phi)$. NOTE: When parameterizing a sphere, ρ is the radius of the sphere, which is a constant!!





$\rho=4$

$\boxed{0\le\theta\le 2\pi}$

To find the upper bound on Φ:

$z=4\cos\Phi$

$2=4\cos\Phi$

$\Phi=\frac{\pi}{3}$

$\boxed{0\le\Phi\le\frac{\pi}{3}}$

$$\iint_S z\,dS=\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\underbrace{4\cos\Phi}_{z=\rho\cos\Phi,\ \rho=4}\ \underbrace{16\sin\Phi}_{\rho^2\sin\Phi,\ \rho=4}\,d\Phi\,d\theta$$

$|r_\Phi\times r_\theta|=16\sin\Phi$

$$=64\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\cos\Phi\sin\Phi\,d\Phi\,d\theta$$

Fubini!

u-sub, $u=\sin\Phi$

$du=\cos\Phi\,d\Phi$

$\int u\,du=\frac{u^2}{2}=\frac{\sin^2\Phi}{2}$

$$=64\int_0^{2\pi}d\theta\int_0^{\frac{\pi}{3}}\cos\Phi\sin\Phi\,d\Phi$$

$$=64\cdot\theta\Big|_0^{2\pi}\ \frac{\sin^2\Phi}{2}\Big|_0^{\frac{\pi}{3}}$$

$$=64(\cancel{2}\pi)\cdot\frac{1}{\cancel{2}}\left(\frac{\sqrt{3}}{2}\right)^2=64\pi\left(\frac{3}{4}\right)=\boxed{48\pi}$$

Problem 13. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle xy, 4x^2, yz \rangle$ and S is the surface $z = xe^y$, $0 \le x \le 1$, $0 \le y \le 1$, with upward orientation.

Let $\vec{r}(x,y) = \langle x, y, xe^y \rangle$, $\boxed{0 \le x \le 1,\ 0 \le y \le 1}$ D

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & e^y \\ 0 & 1 & xe^y \end{vmatrix} = \langle -e^y, -xe^y, 1 \rangle$$

$z>0$, upward orientation

$$\iint_S F \cdot dS = \iint_D \vec{F}(\vec{r}(x,y)) \cdot (\vec{r}_x \times \vec{r}_y)\, dA$$

$$= \iint_{\substack{0 \le x \le 1 \\ 0 \le y \le 1}} \langle xy, 4x^2, y \cdot xe^y \rangle \cdot \langle -e^y, -xe^y, 1 \rangle\, dA$$

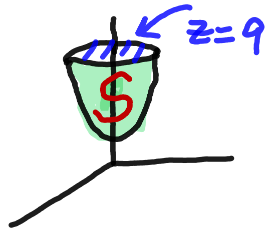
$$= \iint_{\substack{0 \le x \le 1 \\ 0 \le y \le 1}} \left(\cancel{-xye^y} - 4x^3 e^y + \cancel{yxe^y} \right) dA$$

$$= \int_0^1 \int_0^1 (-4x^3 e^y)\, dy\, dx$$ Fubini!! ☺

$$= -4 \int_0^1 x^3\, dx \int_0^1 e^y\, dy = -4 \cdot \frac{x^4}{4} \Big|_0^1 e^y \Big|_0^1 = \boxed{-(e-1)}$$

Problem 14. Set up but do not evaluate a double integral that gives the flux of $\mathbf{F} = \langle\, z-3,\ x,\ y\rangle$ across S, where S is the part of the paraboloid $z = x^2 + y^2 + 3$ that is below the plane $z = 9$. Use the positive (outward) orientation.

$$\text{Flux } F = \iint_S F \cdot dS$$



$$9 = x^2 + y^2 + 3$$
$$6 = x^2 + y^2$$

$$\vec{r}(x,y) = \langle x, y, x^2+y^2+3\rangle, \quad x^2+y^2 \le 6$$

z >0 ✓
positive orientation

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & 2x \\ 0 & 1 & 2y \end{vmatrix} = \langle -2x, -2y, 1\rangle$$

$$\iint_S F \cdot dS = \iint_{x^2+y^2\le 6} \langle x^2+y^2+3-3,\ x,\ y\rangle \cdot \langle -2x, -2y, 1\rangle\, dA$$

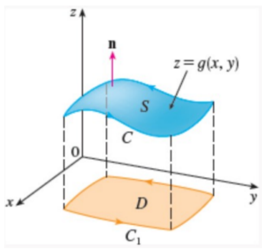
$$= \iint_{x^2+y^2\le 6} \left(-2x(x^2+y^2) - 2xy + y\right) dA$$

convert to polar

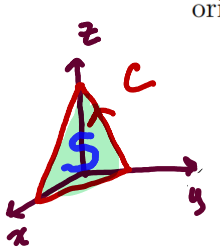
$$= \int_0^{2\pi}\int_0^{\sqrt{6}} \left(-2r\cos\theta(r^2) - 2r^2\cos\theta\sin\theta + r\sin\theta\right) r\,dr\,d\theta$$

Stokes' Theorem: Let S be an oriented piecewise-smooth surface parameterized by $\mathbf{r}(u,v)$, $u,v \in D$, that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive (counterclockwise) orientation. Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\Re^3$ that contains the surface S. Then

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F}\cdot d\mathbf{S}$$



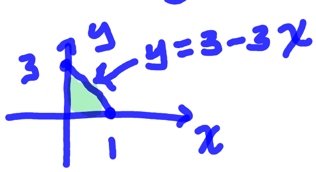
Problem 15. Use Stokes' Theorem to set up but **not evaluate** $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = \langle yz, 2xy, 4xz\rangle$, and where C is the boundary curve of the part of the plane $3x+y+z=3$ in the first octant, oriented counterclockwise when looking from above.



$$\text{curl} F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & 2xy & 4xz \end{vmatrix} = \langle 0, y-4z, 2y-z\rangle$$

Stokes: $\int_C F\cdot dr = \iint_S \text{curl} F \cdot ds$, where C = boundary curve of S.

Since C is ccw, S is the positively oriented plane $3x+y+z=3$ in first octant. If we parameterize S as $\vec{r}(x,y) = \langle x, y, 3-3x-y\rangle$, then D will be in the xy-plane:



D: $0 \le x \le 1$
$0 \le y \le 3-3x$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -3 \\ 0 & 1 & -1 \end{vmatrix} = \langle 3, 1, 1\rangle$$

z>0, positive orientation ✓

$$\iint_S \text{curl} F\cdot ds = \iint_D \text{curl} F(\vec{r}(x,y))\cdot(r_x \times r_y)\, dA$$

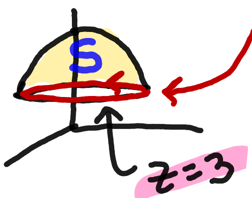
curl F = ⟨0, y−4z, 2y−z⟩

$$= \iint_D \langle 0, \underbrace{y-4(3-3x-y)}_{y-4z}, \underbrace{2y-(3-3x-y)}_{2y-z}\rangle\cdot\langle 3,1,1\rangle\, dA$$

$$= \int_0^1\int_0^{3-3x} \Big(y-4(3-3x-y) + 2y-(3-3x-y)\Big)\, dy\, dx$$

Problem 16. Use Stokes' Theorem to evaluate $\iint_S \text{curl}\,\mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F} = \langle x^2\sin(z-3), y^2, xy\rangle$, and S is the part of the paraboliod $z = 9 - x^2 - y^2$ that lies above the plane $z = 3$, oriented upward.

$\iint_S \text{curl}\, F\cdot dS = \int_C F\cdot dr$, where C = boundary of S.



C = intersection of $z = 9-x^2-y^2$ and $z=3$

$9-x^2-y^2 = 3 \rightarrow \boxed{x^2+y^2=6}$ C which we must parameterize ccw (since S is oriented up)

$C: \vec{r}(\theta) = \langle \sqrt{6}\cos\theta, \sqrt{6}\sin\theta, 3\rangle$, $0 \le \theta \le 2\pi$

$r'(\theta) = \langle -\sqrt{6}\sin\theta, \sqrt{6}\cos\theta, 0\rangle$ $\quad \vec{F} = \langle x^2\sin(z-3), y^2, xy\rangle$

$\int_C F\, dr = \int_0^{2\pi} F(r(\theta))\cdot r'(\theta)\, d\theta$

$= \int_0^{2\pi} \langle \cancelto{0}{6\cos^2\theta\sin(0)}, 6\sin^2\theta, 6\cos\theta\sin\theta\rangle \cdot \langle -\sqrt{6}\sin\theta, \sqrt{6}\cos\theta, 0\rangle\, d\theta$

$= \int_0^{2\pi} 6\sqrt{6}\sin^2\theta\cos\theta\, d\theta$

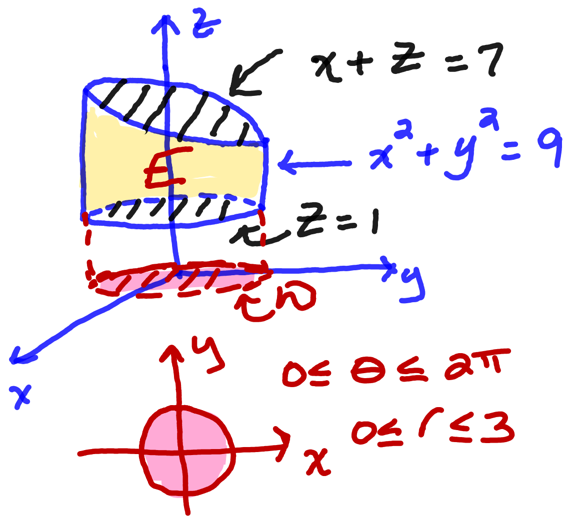
$u = \sin\theta$
$du = \cos\theta\, d\theta$ $\quad \int u^2\, du = \frac{u^3}{3}$

$= 6\sqrt{6}\, \frac{\sin^3\theta}{3}\Big|_0^{2\pi}$

$= 0$

Problem 17. Use the The Divergence Theorem to evaluate $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F} = \left\langle 4x,\ \sin(e^z),\ \sqrt{x^3+y^2}\right\rangle$ and S is the surface bounded by $x^2+y^2=4$, $z=2$, $z=4$.

$$\text{Flux } F = \iint_S F\,dS = \iiint_E \operatorname{div} F\,dV = \iiint_E 2\,dV$$



Define E in cylindrical coordinates

$1 \le z \le 7 - x,\quad x^2 + y^2 \le 9$

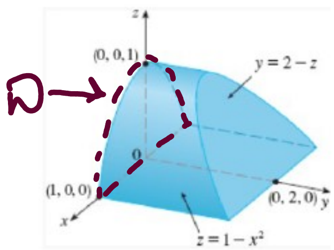
$1 \le z \le 7 - r\cos\theta$

$0 \le \theta \le 2\pi$

$0 \le r \le 3$

$$\boxed{\iiint_E 2\,dV = \int_0^{2\pi}\int_0^{3}\int_1^{7-r\cos\theta} 2\cdot dz\, r\,dr\,d\theta}$$

Problem 18. Using The Divergence Theorem, set up but do not evaluate a triple integral used to find $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F} = \left\langle xy, y^2 + e^{xz^2}, \sin(xy)\right\rangle$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$, and $y + z = 2$. Assume S is positively orientated (see picture below).

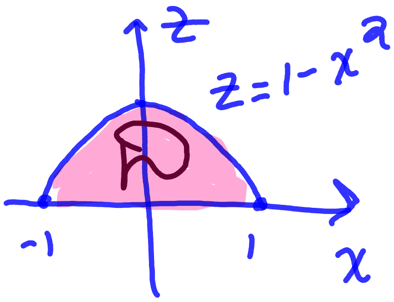


$$\iint_S F\cdot ds = \iiint_E \operatorname{div} F \cdot dV$$

$$= \iiint_E (y + 2y)\, dV$$

$$= \iiint_E 3y\, dV$$

If we choose $dV = dy\, dA$, then $0 \le y \le 2 - z$, and D will be in the xz-plane



$-1 \le x \le 1$

$0 \le z \le 1 - x^2$

E: $0 \le y \le 2 - z$

$0 \le z \le 1 - x^2$

$-1 \le x \le 1$

$$\boxed{\iiint_E 3y\, dV = \int_{-1}^{1}\int_0^{1-x^2}\int_0^{2-z} 3y\, dy\, dz\, dx}$$

Problem 19. Don't forget to review section 14.7, local and absolute extrema. Find all local extrema and/or saddle points if $f(x,y) = y^3 - 6y^2 - 2x^3 - 6x^2 + 48x + 20$.

CP. $f_x(x,y) = -6x^2 - 12x + 48 = -6(x^2 + 2x - 8) = -6(x+4)(x-2)$

$f_y(x,y) = 3y^2 - 12y = 3y(y-4)$ $\quad f_x = 0$ when $x=-4,\ x=2$

$f_y = 0$ when $y=0,\ y=4$

CP: $(-4,0), (-4,4), (2,0), (2,4)$

Test these critical points using the second derivative test: $D = f_{xx} f_{yy} - (f_{xy})^2 = (-12x-12)(6y-12) - 0$

CP	$D = (-12x-12)(6y-12)$	$f_{xx} = -12x-12$	conclusion. $(a,b,f(a,b))=$
$(-4,0)$	$D(-4,0) < 0$	N/A	saddle pt $(-4,0,-44)$
$(-4,4)$	$D(-4,4) > 0$	$f_{xx}(-4,4) > 0$	local min $(-4,4,-172)$ or $z = -172$
$(2,0)$	$D(2,0) > 0$	$f_{xx}(2,0) < 0$	local max $(2,0,76)$ or $z = 76$
$(2,4)$	$D(2,4) < 0$	N/A	saddle pt $(2,4,44)$