# 1 Week 12 HOGU: 5.5-5.8, Exam 3 Review

**Problem 1.** Find the domain of each of the following functions. Draw the domain on the number line, then give your answer using interval notation.

(a) $f(x) = \begin{cases} 3x^2 - 4 & \text{if } x < 8 \\ 7 & \text{if } x = 8 \\ \sqrt[4]{x-4} & x > 8 \end{cases}$

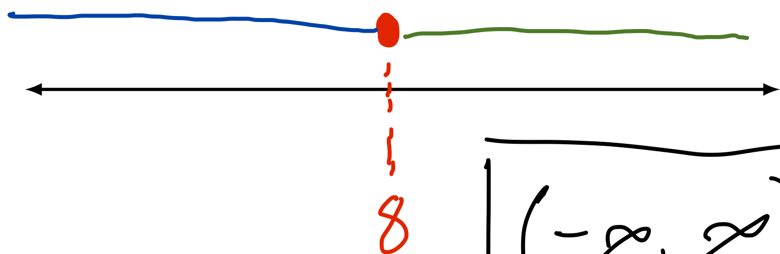
(x<8) Function: No issues! (-∞, 8)

(x=8) Function: No issues! Closed dot at x=8.

(x>8) Function: Even root! Inside ≥ 0

$x - 4 \geq 0$

$x \geq 4$ $\boxed{[4, \infty)}$ (but only when $x > 8$)



$\boxed{(-\infty, \infty)}$

(b) $g(x) = \begin{cases} \dfrac{\sqrt[3]{2x-15}}{x} & \text{if } x \leq 1 \\ \dfrac{9}{\sqrt{x-2}} & \text{if } x > 1 \end{cases}$

(x≤1) Function: denominator ≠ 0!

$x \neq 0$ (odd root is ok)

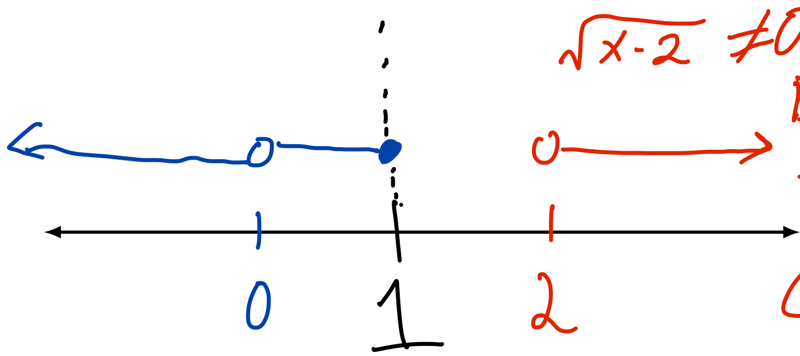
(x>1) Function: denominator ≠ 0!

$\sqrt{x-2} \neq 0 \to x - 2 \neq 0 \to \boxed{x \neq 2}$

Even root: inside ≥ 0!

$x - 2 \geq 0 \to \boxed{x \geq 2}$.

Combine: $\boxed{x > 2}$



$(-\infty, 0) \cup (0, 1] \cup (2, \infty)$

**Problem 2.** Your electric bill came in! On your bill you noticed that you were charged \$7 as a base fee, plus \$6 per kilowatt-hour of electricity used up to the first 100 kilowatt-hours. (These numbers were taken from my own electric bill!) After using 100 kilowatt-hours, you notice that the amount you are charged goes up to \$9 per kilowatt-hour. Construct the piecewise function describing the cost $C(x)$, in dollars, that you pay when using $x$ kilowatt-hours of electricity.

base fee

\$6 per kilowatt-hour

$$C(x) = \begin{cases} 7+6x & \text{if } 0 \le x \le 100 \\ (7+6\cdot 100) + 9(x-100) & \text{if } x > 100 \end{cases}$$

payment for first 100 hours

payment after first 100 hours

| $x$ | $C(x)$ |
|---|---|
| 0 | 7 |
| 1 | $7+6$ |
| 100 | $7+6\cdot 100$ |
| 101 | $(7+6\cdot 100)+9$ |
| 200 | $(7+6\cdot 100)+9(200-100)$ |

A helpful chart!

$$= \begin{cases} 7+6x & \text{if } 0 \le x \le 100 \\ 9x-293 & \text{if } x > 100 \end{cases}$$

**Problem 3.** State the domain of the following functions:

**Rules of Domain**

1) $\frac{1}{\star}$ : $\star \neq 0$!

2) $\sqrt[n]{\star}$ , $n$ even: $\star \geq 0$!

3) $\log_b(\star)$ : $\star > 0$!

(a) $f(x) = 4e^{x-1}$

RoD #1: Ok!

RoD #2: Ok!

RoD #3: Ok!

$(-\infty, \infty)$

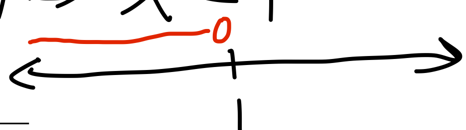
(b) $g(x) = \ln(1-x)$

RoD #1+2: Ok!

RoD #3: $1-x > 0$

$\rightarrow -x > -1 \rightarrow x < 1$

$\boxed{(-\infty, 1)}$



(c) $k(x) = \dfrac{\sqrt{x^3+8}}{\ln(x)}$

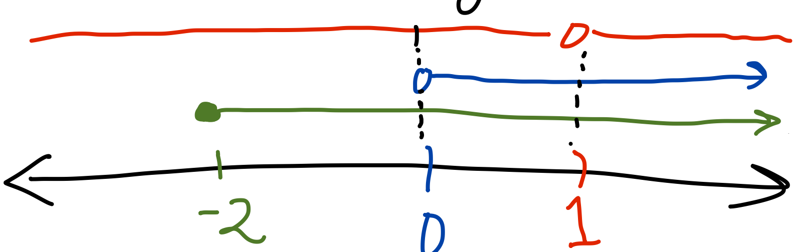
RoD #1: Denominator $\neq 0$! $\ln(x) \neq 0 \rightarrow e^0 \neq x$

$\rightarrow x \neq 1$

RoD #2: Square root inside $\geq 0$! $x^3 + 8 \geq 0 \rightarrow x^3 \geq -8$

$\rightarrow x \geq -2$

RoD #3: Logarithm inside $>0$! $x > 0$



$\boxed{(0,1) \cup (1, \infty)}$

**Problem 4.** (a) Completely simplify this expression to be in base 6:

$$\frac{36^{x^2}}{6^{-4x}}$$

$$\frac{36^{x^2}}{6^{-4x}} = \frac{(6^2)^{x^2}}{\frac{1}{6^{4x}}} = 6^{2\cdot x^2} \cdot \frac{6^{4x}}{1}$$

$$= \boxed{6^{2x^2+4x}}$$

(b) Fully expand the expression using the properties of logarithms:

$$\ln\left(\sqrt[3]{\frac{x^3}{e^2z^4}}\right).$$

$$\ln\left(\left[\frac{x^3}{e^2z^4}\right]^{\frac{1}{3}}\right) = \frac{1}{3}\ln\left(\frac{x^3}{e^2z^4}\right) = \frac{1}{3}\left[\ln(x^3) - \ln(e^2z^4)\right]$$

$$= \frac{1}{3}\left[3\ln(x) - (\ln(e^2) + \ln(z^4))\right] = \frac{1}{3}\left[3\ln(x) - \ln(e^2) - \ln(z^4)\right]$$

$$= \ln(x) - \frac{1}{3}\ln(e^2) - \frac{1}{3}\ln(z^4) = \ln(x) - \frac{2}{3}\ln(e) - \frac{4}{3}\ln(z)$$

$$= \boxed{\ln(x) - \frac{4}{3}\ln(z) - \frac{2}{3}}$$

**Problem 5.** Solve the following equations for $x$:

(a) $4^{x+1} = 64$

• Goal: get both into same base

$4^{x+1} = 4^3$

| $x$ | $4^x$ |
|---|---|
| 1 | 4 |
| 2 | 16 |
| 3 | 64 |

• Set exponents equal

$x+1 = 3 \rightarrow \boxed{x=2}$

(b) $\ln(x) + \ln(x-2) = \ln(x+10)$

$\ln(x(x-2)) = \ln(x+10)$

• Set insides equal

$x(x-2) = x+10 \rightarrow x^2-2x = x+10$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=-2,5$

• Check solutions!

$\ln(5) + \ln(5-2) = \ln(5+10)$

$\ln(5) + \ln(3) = \ln(15)$ ✓

$\ln(-2) + \ln(-2-2) = \ln(-2+10)$

Not defined!

$\boxed{x=5}$

(c) $2 \cdot 3^{-x} = 16$

• x is in exponent... need to take logarithm!

• Get exponent by itself

$\frac{2 \cdot 3^{-x}}{2} = \frac{16}{2}$

$3^{-x} = 8$

Take logarithm

$\ln(3^{-x}) = \ln(8)$

$-x\ln(3) = \ln(8)$

$x = \boxed{\frac{-\ln(8)}{\ln(3)}}$

$-\log_3(8)$ or $\log_3(\frac{1}{8})$ is acceptable!

**Problem 6.** Recall that the accumulated value of an initial deposit, $P$, for $t$ years, at the interest rate $r$ (expressed as a decimal), is

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt},$$

where $m$ represents the number of times the interest is compounded in a year.

If you deposit \$12,000 in this savings account and the interest rate on the account is 7%, how long would it take the savings account to grow to \$25,000? Assume interest is compounded yearly

$$25000 = 12000(1+.07)^t$$

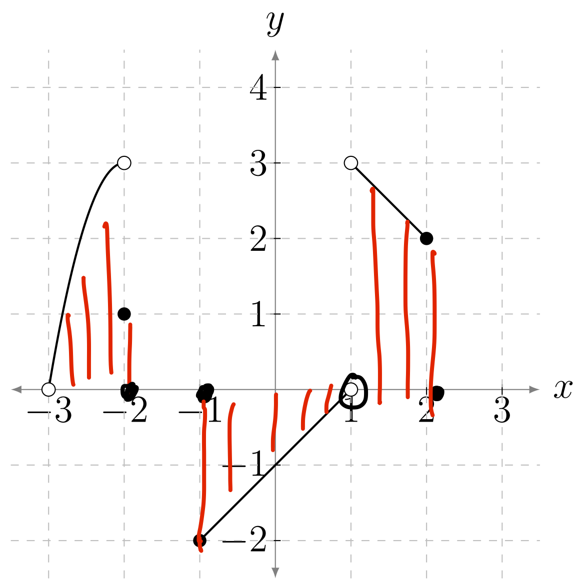
$$\frac{25000}{12000} = 1.07^t$$

- t is in exponent! Take logarithms

$$\ln\left(\frac{25}{12}\right) = \ln(1.07)^t = t\ln(1.07)$$

$$\boxed{\frac{\ln\left(\frac{25}{12}\right)}{\ln(1.07)} = t} \approx 10.848 \text{ years}$$

**Problem 7.** Consider the function $f(x)$ below:




(a) State the domain of $f(x)$. Write your answer in interval notation.

$(-3,-2] \cup [-1,1) \cup (1,2]$

(b) State the range of $f(x)$. Write your answer in interval notation.

$[-2,0) \cup (0,3)$

**Problem 8.** Compute and completely simplify the difference quotient for the function $g(x) = -\frac{3}{x+1}$.

(a) $g(x+h) = -\frac{3}{(x+h)+1}$

(b) $g(x+h) - g(x) =$

$$\frac{-3}{(x+h)+1} - \frac{-3}{x+1}$$

Least common denominator? $[(x+h)+1] \cdot [x+1]$

$$\frac{(x+1)(x+h+1)}{x+h+1} \qquad \frac{-3(x+1) + 3(x+h+1)}{(x+1)(x+h+1)} \qquad \frac{(x+1)(x+h+1)}{x+1}$$

new denominator

old denominator

$$= \frac{-\cancel{3x}-\cancel{3}+\cancel{3x}+3h+\cancel{3}}{(x+1)(x+h+1)} = \frac{3h}{(x+1)(x+h+1)}$$

(c) $\frac{g(x+h)-g(x)}{h} =$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{3h}{(x+1)(x+h+1)}}{\frac{h}{1}} = \frac{3\cancel{h}}{(x+1)(x+h+1)} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{3}{(x+1)(x+h+1)}}$$

Multiply by reciprocal!

**Problem 9.** Compute and completely simplify the difference quotient for the function $k(x) = \sqrt{2x-5}$.

(a) $k(x+h) =$ $\sqrt{2(x+h)-5} = \sqrt{2x+2h-5}$

Rationalize the numerator

(b) $k(x+h) - k(x) =$

$$\frac{\sqrt{2x+2h-5}-\sqrt{2x-5}}{1} \cdot \frac{\sqrt{2x+2h-5}+\sqrt{2x-5}}{\sqrt{2x+2h-5}+\sqrt{2x-5}}$$

Conjugate on top and bottom: flip the sign!

$$= \frac{(2x+2h-5)-(2x-5)}{\sqrt{2x+2h-5}+\sqrt{2x-5}} = \frac{\cancel{2x}+2h-\cancel{5}-\cancel{2x}+\cancel{5}}{\sqrt{2x+2h-5}+\sqrt{2x-5}}$$

$$= \frac{2h}{\sqrt{2x+2h-5}+\sqrt{2x-5}}$$

(c) $\dfrac{k(x+h)-k(x)}{h} =$

$$\frac{\dfrac{2h}{\sqrt{2x+2h-5}+\sqrt{2x-5}}}{\dfrac{h}{1}} = \frac{2\cancel{h}}{\sqrt{2x+2h-5}+\sqrt{2x-5}} \cdot \frac{1}{\cancel{h}} = \frac{2}{\sqrt{2x+2h-5}+\sqrt{2x-5}}$$