MATHEMATICS
TEXAS A&M UNIVERSITY

Week-in-Review#7 - Exam 2 Review
Covers Sections 2.3-2.6, 3.1-3.3
Math 142 - Fall 2024, by K. Kilmer

the second derivative

$\frac{d}{dx}(5x^{-1}) = 5\cdot -1x^{-1-1}$

1. Given $f(x) = \frac{5}{x} + x\ln(5x-3)$ find $f''(x)$.

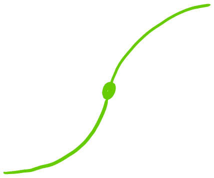
$= \underbrace{5x^{-1}} + \underbrace{\underbrace{x}_{F}\underbrace{\ln(5x-3)}_{S}}$

$f'(x) = -5x^{-2} + \underbrace{\frac{x}{1}}_{F}\cdot\underbrace{\frac{1}{5x-3}\cdot\frac{5}{1}}_{S'} + \underbrace{\ln(5x-3)}_{S}\cdot\underbrace{1}_{F'} = \underbrace{-5x^{-2}} + \frac{\overset{T}{5x}}{\underbrace{5x-3}_{B}} + \boxed{\underbrace{\ln(5x-3)}}$ ← 1st Derivative

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

$\frac{d}{dx}(\ln(f(x))) = \frac{1}{f(x)}\cdot f'(x) = \frac{f'(x)}{f(x)}$

$$\boxed{f''(x) = 10x^{-3} + \frac{\overset{B}{(5x-3)}\overset{T'}{(5)} - \overset{T}{(5x)}\overset{B'}{(5)}}{\underbrace{(5x-3)}_{B}^2} + \frac{1}{5x-3}\cdot 5}$$



2. If the domain of $f(x)$ is $(-\infty,-3)\cup(-3,\infty)$ and $f''(x) = \frac{(x+5)(x-2)^2}{x+3}$ y_1, determine the intervals on which $f(x)$ is concave up and concave down and determine where the inflection points occur.

⇒ create a sign chart of $f''(x)$

① We first find the partition #'s of $f''(x)$:

$f''(x) = 0$ when $(x+5)(x-2)^2 = 0$

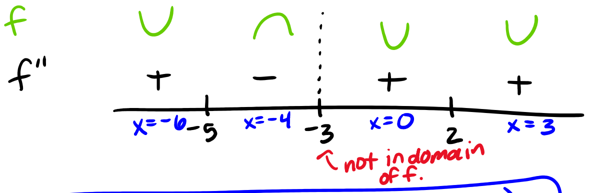
$x+5=0$ $\quad (x-2)^2=0$

$x=-5$ $\quad x-2=0$

$x=2$

$f''(x)$ DNE when $x+3=0$

$x=-3$



Concave Up $(-\infty,-5), (-3,2), (2,\infty)$
Concave Down $(-5,-3)$
Inflection Point at $x=-5$

3. Use the second derivative test for local extrema to find (and classify) the local extrema of $f(x) = \frac{2}{3}x^3 + \frac{5}{2}x^2 - 12x + 20.$ Domain: $(-\infty,\infty)$

① Find the critical values of $f(x)$:

$f'(x) = \underbrace{2x^2} + \underbrace{5x} \underbrace{-12}$

$f'(x) = 0$ when $2x^2+5x-12=0$

$(2x-3)(x+4)=0$

$2x-3=0$ $\quad x+4=0$

$2x=3$ $\quad x=-4$

$x=3/2$

critical values

check:
$(2x-3)(x+4)$
$= 2x^2+8x-3x-12$
$= 2x^2+5x-12$ ✓

② Evaluate $f''(x)$ at $x=-4$ and $x=3/2$:

$f''(x) = 4x+5$

$\underline{x=-4}$

$f''(-4) = 4(-4)+5 = -$ ∩ ⇒ Local max of $\underline{196/3}$ at $x=-4$.

$\underline{x=3/2}$

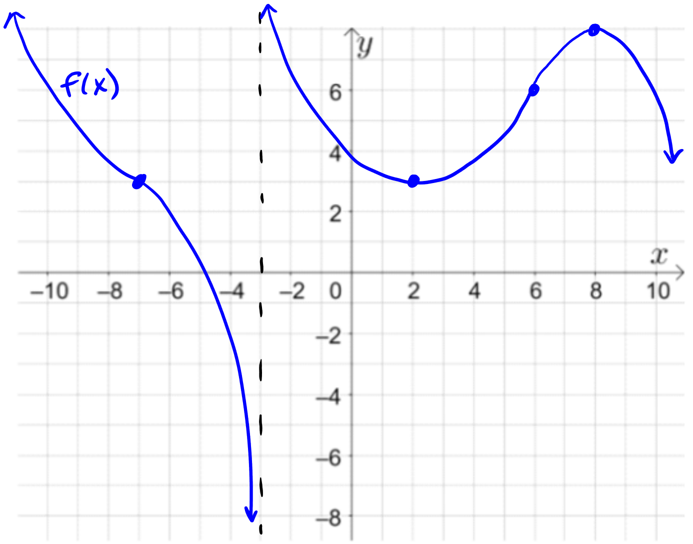
$f''(3/2) = 4(3/2)+5 = +$ ∪ ⇒ Local min of $\underline{79/8}$ at $x=3/2$.

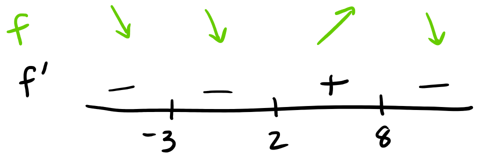
f(−4)

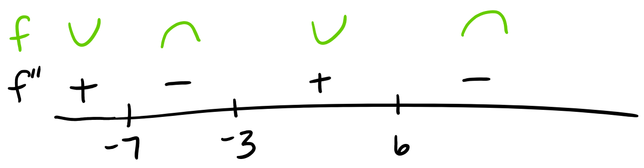
f(3/2)

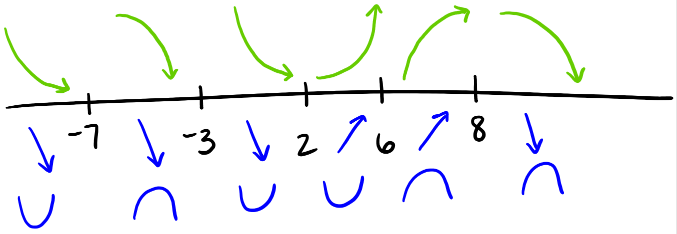
4. Sketch a graph of $f(x)$ given the following information:

- Domain of $f(x)$ is $(-\infty,-3)\cup(-3,\infty)$
- ✓ $f(-7)=3,\ f(2)=3,\ f(6)=6,\ f(8)=8$
- ✓ $x=-3$ is a vertical asymptote
- ✓ $f'(x)>0$ on $(2,8)$
- ✓ $f'(x)<0$ on $(-\infty,-3),(-3,2),(8,\infty)$
- ✓ $f''(x)>0$ on $(-\infty,-7),(-3,6)$
- $f''(x)<0$ on $(-7,-3),(6,\infty)$









5. Find the equation of the line tangent to the graph of $f(x)=\underline{3x^2}+\underline{e^x}-\underline{4\ln x}$ at $x=1$.

① Find $f'(x)$:

$f'(x)=6x+e^x-4\cdot\frac{1}{x}$

② Find $f'(1)$ to find the slope of the line:

$f'(1)=6(1)+e^1-4\cdot\frac{1}{1}=6+e-4=2+e$ ← slope of the tangent line

③ Find $f(1)$ to find the y-coord. of the point:

$f(1)=3(1)^2+e^1-4\underbrace{\ln(1)}_{0}$

$=3+e-0$

$=3+e$

④ Find the equation:

$(1, 3+e)$ $m=2+e$

$y=mx+b$

$3+e=(2+e)(1)+b$

$3+e=\cancel{2+e}+b$

$-2-e\quad \cancel{-2-e}$

$1=b$

$\boxed{y=(2+e)x+1}$

Marginal Analysis (sec. 2.3)

6. The profit function for a company that makes and sells backpacks is given by $P(x) = -0.2x^2 + 460x - 8000$, where $P(x)$ is the profit in dollars when x backpacks are made and sold.

⇒ use the derivative!

(a) Estimate the profit from making and selling the $\underbrace{1{,}125}_{n}$th backpack.

$P'(x) = -0.4x + 460$

$P'(n-1) = P'(1124) = -0.4(1124) + 460 = \$10.40$/backpack

⇒ The profit from the 1,125th backpack is approx \$10.40

⇒ use the original function

(b) Find the exact profit from making and selling the $\underbrace{1{,}125}_{n}$th backpack.

$P(n) - P(n-1) = P(1125) - P(1124)$

$= 256375 - 256364.80 = \boxed{\$10.20}$

7. If $f(x) = \dfrac{x(2^x-4)^3}{\log_3(8-7x^2)}$, what is $f'(x)$?

$T = \underbrace{x}_{F}\underbrace{(2^x-4)^3}_{S}$

$T' = \underbrace{x}_{F} \cdot \underbrace{3(2^x-4)^2 \cdot \ln 2 \cdot 2^x}_{S'} + \underbrace{(2^x-4)^3}_{S} \cdot \underbrace{1}_{F'}$

$B = \log_3(8-7x^2)$

$B' = \frac{1}{\ln 3} \cdot \frac{1}{8-7x^2} \cdot -14x$

$$\boxed{f'(x) = \frac{\overset{B}{\left(\log_3(8-7x^2)\right)}\overset{T'}{\left(x \cdot 3(2^x-4)^2 \cdot \ln 2 \cdot 2^x + (2^x-4)^3\right)} - \overset{T}{\left(x(2^x-4)^3\right)}\overset{B'}{\left(\frac{1}{\ln 3} \cdot \frac{1}{8-7x^2} \cdot -14x\right)}}{\underset{B}{\left(\log_3(8-7x^2)\right)^2}}}$$

$\frac{d}{dx}\left(\log_3(x)\right) = \frac{1}{\ln 3} \cdot \frac{1}{x}$

$= \frac{1}{\ln 3 \cdot x} = \frac{1}{x \cdot \ln 3}$

$\frac{d}{dx}\left(\log_3(f(x))\right) = \frac{1}{\ln 3} \cdot \frac{1}{f(x)} \cdot f'(x)$

8. If $f(2) = 2$, $f'(2) = -1$, $f(4) = 0$, $f'(4) = 1$, $g(2) = 3$, $g'(2) = 5$, and $h(x) = \dfrac{4f(x^2)}{x \cdot g(x)}$, what is $h'(2)$?

$T(x) = 4f(x^2)$

$T'(x) = 4 \cdot f'(x^2) \cdot 2x$

$B(x) = x \cdot g(x)$

$B'(x) = x \cdot g'(x) + g(x) \cdot 1$

$h'(x) = \frac{B(x) \cdot T'(x) - T(x) \cdot B'(x)}{(B(x))^2}$

when x=2:

$T(2) = 4f(2^2) = 4f(4) = 4(0) = \underline{0}$

$T'(2) = 4 \cdot f'(2^2) \cdot 2 \cdot 2 = 16 \cdot f'(4) = 16(1) = \underline{16}$

$B(2) = 2 \cdot g(2) = 2(3) = \underline{6}$

$B'(2) = 2 \cdot g'(2) + g(2) = 2(5) + 3 = \underline{13}$

$h'(2) = \frac{B(2) \cdot T'(2) - T(2) \cdot B'(2)}{(B(2))^2} = \frac{(6)(16) - (0)(13)}{(6)^2}$

$= \frac{96}{36} = \boxed{\frac{8}{3}}$

Let's pretend $y = 5x^2+4$

$\frac{d}{dx}(y^3) = \frac{d}{dx}\left((5x^2+4)^3\right)$

$= 3\underbrace{(5x^2+4)}_{y}^2 \cdot \underbrace{10x}_{\frac{dy}{dx}}$

$= 3y^2 \cdot \frac{dy}{dx}$

Use implicit differentiation!

9. Find $\frac{dy}{dx}$ if $x^2y^3 + 2x^3 - 4y^2 = x$.

$\frac{d}{dx}\left(\underbrace{x^2}_{F}\underbrace{y^3}_{S} + \underbrace{2x^3} - \underbrace{4y^2}\right) = \frac{d}{dx}(x)$

$\underbrace{x^2 \cdot 3y^2 \cdot \frac{dy}{dx}} + \cancel{y^3 \cdot 2x} + \cancel{6x^2} - \underbrace{8y \cdot \frac{dy}{dx}} = 1$

$\cancel{-2xy^3} \quad \cancel{-6x^2} \qquad -2xy^3 - 6x^2$

$3x^2y^2 \cdot \frac{dy}{dx} - 8y \cdot \frac{dy}{dx} = 1 - 2xy^3 - 6x^2$

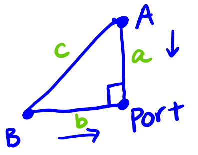
$\frac{dy}{dx}\left(3x^2y^2 - 8y\right) = 1 - 2xy^3 - 6x^2$

$$\boxed{\frac{dy}{dx} = \frac{1-2xy^3-6x^2}{3x^2y^2-8y}}$$

Ⓐ Related Rates!

10. A Ⓐ ship is observed to be 5 miles due north of port and travelling due south at 2 miles per hour. At the same time, another Ⓑ ship is observed to be 12 miles due west of port and travelling due east on its way back to port at 3 miles per hour. What is the rate at which the distance between the ships is changing at that time?

①



② $a = 5$ miles

$\frac{da}{dt} = -2$ miles/hour

$b = 12$ miles

$\frac{db}{dt} = -3$ miles/hour

$\frac{dc}{dt} = ?$

$c = 13$ miles

We still need c:

$a^2+b^2=c^2$

$5^2+12^2=c^2$

$169=c^2$

$c=13$

③ $a^2+b^2=c^2$

④ $\frac{d}{dt}(a^2+b^2) = \frac{d}{dt}(c^2)$

$2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt} = 2c \cdot \frac{dc}{dt}$

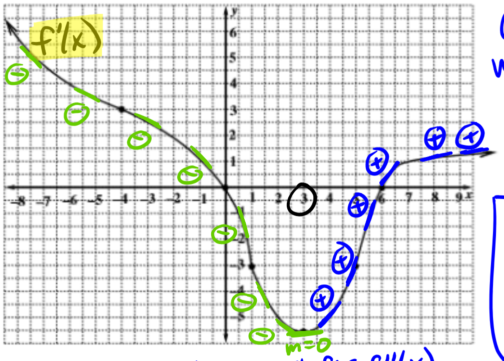
⑤ $2(5)(-2) + 2(12)(-3) = 2(13) \cdot \frac{dc}{dt}$

$-92 = 26 \cdot \frac{dc}{dt}$

$\frac{dc}{dt} = \frac{-92}{26} = \frac{-46}{13}$

The distance between the ships is decreasing by $\frac{46}{13}$ miles/hour

11. Given the graph of $f'(x)$ below and the domain of $f(x)$ is $(-\infty,\infty)$, find (a) the intervals on which $f(x)$ is increasing/decreasing, (b)the x-value for which $f(x)$ has local extrema (and classify), (c) the intervals on which $f(x)$ is concave up/concave down, and (d)the x-value for which $f(x)$ has inflection points.



(a) & (b) ⇒ Create a sign chart of $f'(x)$

We are looking at whether it is above or below the x-axis.

f: ↗ ↘ ↗

f': + | − | +

0 (CV), 6 (CV)

a) f is increasing on $(-\infty, 0), (6, \infty)$

f is decreasing on $(0, 6)$

b) Local Max at $x=0$

Local Min at $x=6$

(c) & (d) ⇒ Create a sign chart for $f''(x)$.

Since f'' is the derivative of this graph, we are looking at slopes.

f: ∩ ∪

f'': − | +

3

c) f is concave down on $(-\infty, 3)$

f is concave up on $(3, \infty)$

d) Inflection Point at $x=3$