REVIEW OF ALGEBRA

- Simplifying Fractions
- Order of Operations
- Multiplying Expressions
- Factoring

Pr 1. Compute each of the following and simplify completely.

(a) $\frac{5}{6} \cdot 30 = \frac{5}{6} \cdot \frac{30}{1} = \frac{5 \cdot \cancel{30}^{5}}{\cancel{6} \cdot 1} = \frac{150}{6}$ ← not simplified $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

$= \frac{5 \cdot 5}{1} = \boxed{25}$

(b) $-15 \div \frac{1}{6} = \frac{-15}{1} \div \frac{1}{6} = \frac{-15}{1} \times \frac{6}{1} = \frac{-15 \times 6}{1 \times 1} = \boxed{-90}$

$-\frac{a}{b} = \frac{-a}{b}$

(c) $-\frac{11}{10} + \frac{27}{15} = \frac{15}{15} \cdot \frac{-11}{10} + \frac{10 \cdot 27}{10 \cdot 15}$ $\frac{a}{b} + \frac{c}{d}$ ← common denominator

$= \frac{-165}{150} + \frac{270}{150} = \frac{-165+270}{150} = \frac{105}{150} = \frac{21}{30} = \boxed{\frac{7}{10}}$

$$\begin{array}{r} 270 \\ -165 \\ \hline 105 \end{array}$$

(d) $\frac{9}{5} - \left(-\frac{7}{9}\right) = \frac{9}{5} + \frac{7}{9} = \frac{9 \cdot 9}{9 \cdot 5} + \frac{5 \cdot 7}{5 \cdot 9}$

$= \frac{81+35}{45} = \boxed{\frac{116}{45}}$

(e) $\frac{5}{20} \cdot \frac{49}{10} \cdot \frac{20}{5} =$ simplify first

$\frac{\cancel{5} \cdot 49 \cdot \cancel{20}}{\cancel{20} \cdot 10 \cdot \cancel{5}} = \boxed{\frac{49}{10}}$

(f) $\frac{3}{5} + \frac{5}{9} + \left(-\frac{3}{5}\right) = +\cancel{\frac{3}{5}} + \frac{5}{9} - \cancel{\frac{3}{5}} = \boxed{\frac{5}{9}}$

Pr 2. Compute each of the following and simplify completely.

$a^{-6} = \frac{1}{a^6}$

$\frac{1}{a^{-6}} = a^6$

(a) $\boxed{\frac{1}{3^{-4}}} = 3^4 = 3\times3\times3\times3 = 9\times9 = \boxed{81}$

$\neq 3\times4$

(b) $\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^{+2} = \frac{3^2}{5^2} = \boxed{\frac{9}{25}}$

reciprocal

$\left(\frac{a}{b}\right)^{-c} = \frac{1}{\left(\frac{a}{b}\right)^c} = \left(\frac{b}{a}\right)^c$

$a^b \cdot a^c = a^{b+c}$

(c) $4^5 \cdot 4^3 = \underbrace{4\times4\times4\times4\times4}\times\underbrace{4\times4\times4}$

$= 4^{5+3} = 4^8 = \boxed{65536}$

$4\times8 = 32$

big difference

(d) $-\sqrt{81} = -(81)^{1/2}$

$= -9$

$\sqrt{81} = x \rightarrow x^2 = 81$

(e) $\sqrt{-64}$ = Does not exist

$\sqrt{\text{negatives}}$ are not real numbers

(f) $\sqrt[3]{-8} = -2$ ✓

$\sqrt[3]{-8} = x, \quad x^3 = -8$

$(-2)\cdot(-2)\cdot(-2) = -8$

odd root $\sqrt{\text{negative number}}$ does exist

Pr 3. Simplify the expression, using the order of operations.

P E MD AS

(a) $4(2+8\cdot4) - 7^2 = 4(2+32) - 7^2$

$= 4\times34 - 49$

$= 136 - 49 = \boxed{87}$

(b) $5^2 - 16 \div (9-5) = 5^2 - 16 \div 4$

$= 25 - 16 \div 4$

$= 25 - 4 = \boxed{21}$

(c) $(x-y)^2$ when $x = 9, y = 7$

$(9-7)^2 = 2^2 = 4$

$9^2 - 7^2 = 81 - 49$

$= 32$

not equal

$(x-y)^2 \neq x^2 - y^2$

$(x+y)^2 \neq x^2 + y^2$

$\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$

Pr 4. Simplify each of the following.

(a) $3b^2 - 7b + 10 + 2b^2 + 3b - 4$ $= 3b^2 + 2b^2 - 7b + 3b + 10 - 4$

$= (3+2)b^2 + (-7+3)b + 6$

$= \boxed{5b^2 - 4b + 6}$

(b) $(3x)^2(5x)$ $= 3^2 \cdot x^2 \cdot (5x)$ not $3 \cdot x^2$

$(ab)^c = a^c \cdot b^c$

$= 9 \cdot x^2 \cdot 5 \cdot x = 9 \cdot 5 \cdot x^2 \cdot x$

$= \boxed{45 \cdot x^3}$, not $15x^3$

(c) $\left(\frac{1}{3}f^7\right)(18f^3)$ $= \frac{1}{3} \cdot 18 \cdot f^7 \cdot f^3$

$= \frac{18}{3} \cdot f^{7+3}$

$= \boxed{6 \cdot f^{10}}$

$a(b+c) = ab + ac$

(d) $5q^3(q^2 - q + 5)$

$= 5q^3 \cdot q^2 + 5q^3(-q) + 5q^3 \cdot 5 = 5q^{3+2} - 5q^{3+1} + 5 \cdot 5q^3$

$= \boxed{5q^5 - 5q^4 + 25q^3}$

F.O.I.L

(e) $(q+3)(q-6)$ $= q \cdot q + q(-6) + 3q + (3)(-6)$

$= q^2 - 6q + 3q - 18 = \boxed{q^2 - 3q - 18}$

(f) $(x+5)(x^2+3x-4)$ $= x \cdot x^2 + x \cdot 3x + x(-4)$
$+ 5 \cdot x^2 + 5 \cdot 3x + 5 \cdot (-4)$

$x^3 + 3x^2 - 4x$
$\to 5x^2 + 15x - 20$

$x^3 + 8x^2 + 11x - 20$

F.O.I.L

(g) $(4-5y)(4+5y)$ $= 4 \times 4 + 4 \cdot 5y + (-5y) \cdot 4 + (-5y)(5y)$

$= 16 + 20y - 20y - 25y^2$

$= \boxed{16 - 25y^2}$

$(a-b)(a+b) = a^2 - b^2$

$4^2 - (5y)^2$

useful formula for 5.4

(h) $\dfrac{(y+5)(3-y)}{(y+4)(y-3)}$ hard way = Foil the numerator + denominator

$3-y$ vs $y-3$

$-(a-b) = -a + b$

$-(y-3) = -y + 3$
$= 3-y$

$= \dfrac{(y+5)(-1)\cancel{(y-3)}}{(y+4)\cancel{(y-3)}}$ "=" $\boxed{\dfrac{-y-5}{y+4}}$

Pr 5. Factor each of the following. ← find a common factor

(a) $4x^3 - 12x^2 + 16x$ $= x(4x^2 - 12x + 16)$ $= 4x(x^2 - 3x + 4)$ $= \boxed{4x(x^2 - 3x + 4)}$ → See Appendix of text book

$x^3 = x \cdot x^2$
$x^2 = x \cdot x$

$12 = 4 \times 3$
$16 = 4 \times 4$

$4 = a \cdot b$
$a + b = -3$

4: $-4, -1$; 4: $-2, -2$

check
$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 4}}{2}$
$= \frac{3 \pm \sqrt{9 - 16}}{2}$
no real root

(b) $8x^3 - 8x^2 + 8x - 8$
$= 8(x^3 - x^2 + x - 1)$
$= 8(x^2(x-1) + 1(x-1)) = \boxed{8(x-1)(x^2+1)}$

(c) $m^2 - 11m + 30$ $= (m - 5)(m - 6)$

check = Foil
$m^2 - 6m - 5m + (-5)(-6)$

$+30$: $-a, -b$; $-a + -b = -11$
30: $-5, -6$; $-5 + -6 = -11$

(d) $r^2 - 4r - 12 = (r-6)(r+2)$

-12: a, b; $a + b = -4$ ← at least one negative

12: $12, -1$ → 11; 12: $-6, +2$ → -4; 12: $4, -3$ → 1

(e) $6w^2 - w - 15$ $= (3w-5)(2w+3)$

rational roots test...

$(6w + a)(w + b)$

Try: $(3w-5)(2w+3)$ or $(3w + a)(2w + b)$
$9w - 10w = -w$

-15: $-, +$

$15 = 15 \cdot 1$ or $= 5 \cdot 3$

(f) $q^3 - 10q^2 - 24q$
$= q(q^2 - 10q - 24)$
$= \boxed{q(q+2)(q-12)}$

24: $-1, 24$ → 23; 24: $-(-2), -(12)$ → -10; 24: $-3, 8$ ...

(g) $98r^3 - 72r$
$= r(98r^2 - 72) = 2r(49r^2 - 36)$
$= 2r((7r)^2 - 6^2)$ ← differences of squares
$= 2r(7r+6)(7r-6)$

$98 = 2 \cdot 49$
$72 = 2 \cdot 36$
$= 2^2 \cdot 18$
$= 2^3 \cdot 9$

(h) $8p^2 + 2$ ← doesn't factor with real numbers → $x^2 +$ positive real

$8p^2 + 2 = 0$
$8p^2 = -2$
$p^2 = \frac{-2}{8} = \frac{-1}{4}$
$p = \pm\sqrt{-1/4}$ ← not real

Section 5.1 Part A: Writing Interval Notation

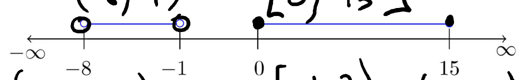
- Set-builder notation
- Interval notation
- Segment of the real number line
- Verbal description

(← open parentheses

[← square bracket

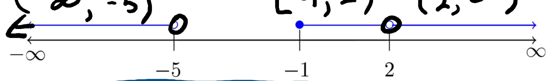
Pr 1. Express each of the following using equivalent interval notation and then give a verbal description for each interval.

(a) $(-8,-1)$ $[0,15]$



$\boxed{(-8,-1) \cup [0,15]}$

(b) $(-\infty,-5)$ $[-1,2)$ $(2,\infty)$



$\boxed{(-\infty,-5)\cup[-1,2)\cup(2,\infty)}$

(c) $\left\{x \mid x < \frac{1}{3}\right\}$ $= \boxed{\left(-\infty, \frac{1}{3}\right)}$ no lower bound

simplify

(d) $\{x \mid x \leq -4 \text{ and } x > 10\}$ → $(-\infty,-4] \cap (10,\infty) = \boxed{\emptyset}$

(e) $\{x \mid x \neq \pm 4\}$ $= \boxed{(-\infty,-4)\cup(-4,4)\cup(4,\infty)}$

-4 4

$(-\infty,-4)\cup(-4,4)\cup(4,\infty)$

Pr 2. State the inputs and outputs of each relation.

(a) $R_1 = \{(10,-23),\ (-8,41),\ (-12,41),\ (36,36)\}$

inputs $= \{-12,-8,10,36\}$

outputs $= \{-23,36,41\}$

(b) $R_2 = \{(-200,-450),\ (-450,-200),\ (375,-375),\ (-450,270)\}$

inputs $= \{-450,-200,375\}$ outputs $= \{-450,-375,-200,270\}$ only once

Pr 3. Determine if the given relation is a function. If the relation is a function, state the domain and range of the function.

no duplicate inputs

(a) $R_3 = \{(10,-23),\ (-8,41),\ (-12,41),\ (36,36)\}$ is a function

domain $= \{-12,-8,10,36\}$

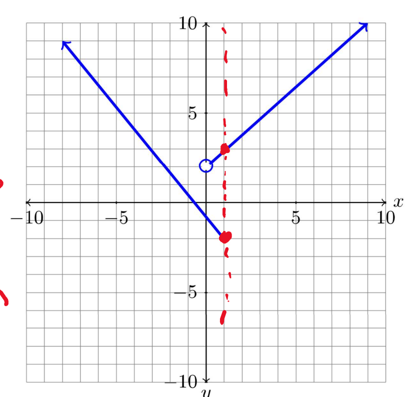
range $= \{-23,36,41\}$

(b) $3x+y=15$ yes domain = range $= (-\infty,\infty)$

$\{(x,y) \mid 3x+y=15\}$

$y=-3x+15$

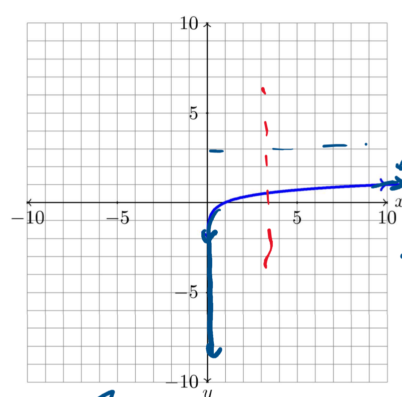
(c)



Not a function

$(1,-2)$, $(1,3)$ are in the relation

(d)



passes the vertical line test ✓

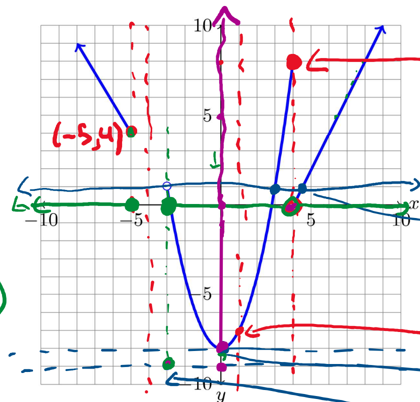
is a function.

domain: $(0,\infty)$

range: $(-\infty,\infty)$

$y=\log x$

Pr 4. Use the graph of $g(x)$ below to answer each of the following.



(4, 8)

y = 1

(3,1) and (4.5,1)

(1,7)

(0,-8)

(-3,-9)

domain gap between -5 and -3

$(-\infty,-5]\cup[-3,\infty)$

range:

$[-8,\infty)$

$\cup\ \{-9\}$

(a) $g(-4)$ = Does not exist

(b) $g(1)$ = 7

(c) $g(4)$ = 8

(d) $g(-5)$ = 4

(e) $g(x) = 1$ y=1 $g(x)=1$ when $x = 3$ or $x = 4.5$

solve $g(x)=y$

(f) $g(x) = -8$ y = -8 when $x = 0$

(g) $g(x) = -9$ y = -9 when $x = -3$

(h) State the domain of $g(x)$. $(-\infty,-5]\cup[-3,\infty)$

(i) State the range of $g(x)$. $\{-9\}\cup[-8,\infty)$

or $[-9,-9]\cup[-8,\infty)$

x input, C(x) is the output

Pr 5. Suppose that we let $C(x)$ represent the total cost of making x fidget spinners.

(a) What does $C(100) = 50$ mean, in the context of this problem?

input= 100, out put = 50

The total cost of making 100 fidget spinners is \$ 50.

input=25 output

(b) How would we represent the expression 'the total cost for making 25 fidget spinners is \$ 10' using function notation?

$C(25) = 10.$

Pr 6. Use the function $f(x) = 3 - 2x$ to compute and expand and simplify each of the following.

(a) $f(8) = 3 - 2(8) = 3 - 16 = \boxed{-13}$

replace x with 8

(b) $f(3q) = 3 - 2(3q) = \boxed{3 - 6q}$

replace x with 3q

(c) $f(x - 5) = 3 - 2(x-5) = 3 - 2x + (-2)(-5)$

$= 3 - 2x + 10$

$= \boxed{-2x + 13}$

$= \boxed{13 - 2x}$

replace x with (x-5)

not $3 - 2x - 5$

(d) $f(x) - f(5) = (3 - 2x) - (3 - 2(5))$

$= 3 - 2x - (3 - 10)$

$= 3 - 2x - (-7)$

$= 3 - 2x + 7$

$\boxed{= 10 - 2x}$

future: simplify $\dfrac{f(x+h) - f(x)}{h}$ for Math 142

Pr 7. Use the function $g(x) = 4x^2 - 3x$ to compute and expand and simplify each of the following.

(a) $g(4) = 4(4)^2 - 3(4) = 4 \cdot 16 - 12$

replace x with (4)

$= 64 - 12$

$= \boxed{52}$

(b) $g(x+h) = 4(x+h)^2 - 3(x+h) = 4(x^2 + 2xh + h^2) - 3(x+h)$

replace x with $(x+h)$

$= \boxed{4x^2 + 8xh + 4h^2 - 3x - 3h}$

$(x+h)^2 = x^2 + xh + hx + h^2$

$= x^2 + 2xh + h^2$

(c) $g(x+h) - (g(x)) = 4x^2 + 8xh + 4h^2 - 3x - 3h - (4x^2 - 3x)$

don't forget the parentheses

$4x^2 + 8xh + 4h^2 - 3x - 3h - 4x^2 + 3x$

$= 8xh + 4h^2 - 3h$

$= h(8x + 4h - 3)$